Analysis of Fixed point theory on Currency Distribution

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Abstract: In this paper we analysis a fixed point theory on currency distribution on population which we are examined to applying market equilibrium & Fredholm integral equation.

Keywords : Banach contration ,Lipschitz condition ,Market equilibrium, Fredholm Integral equation & Fixed point .

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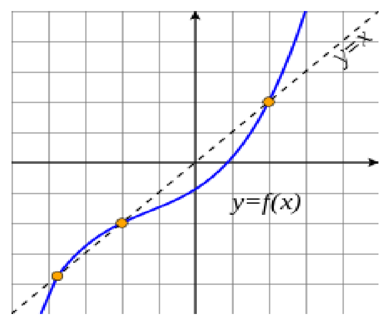
I. INTRODUCTION

1.Fixed Point

In mathematics a Fixed Point of a function is an element of the function's domain that is mapped to itself by the function. That is to say, c is a fixed point of the function $f(x)$ if and only if $f(c) = c$. This means $f(f(...f(c)...)) = f^n(c) = c$, an important terminating consideration when recursively computing f. A set of fixed points is sometimes called a *fixed set*.



For example, if f is defined on the real numbers by

$$F(x)=x^2 - 3x + 4$$

then 2 is a fixed point of f, because $f(2) = 2$.

II. LIPSCHITZ MAPPING

In 1922 Banach published his fixed point theorem also known as Banach's Contraction Principle uses the concept of Lipschitz mapping

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Definition 2.1: Let (X, d) be a metric space. The map $T: X \to X$ is said to be Lipschitzian if there exists a constant $\sigma(T) > 0$ (called Lipschitz constant) such that

$$\boldsymbol{d(T(x), T(y) \leq \sigma(T) d(x, y) \forall\ x, y \in X}$$

(Banach Contraction Principle) **Definition 2.2:** A Lipschitzian mapping with Lipschitzian constant σ ( ) < 1 is called contraction.

Let (X, d) be a complete metric space and $T: X \to X$ be a contraction mapping. Then T has a unique fixed point x_0 and for each $x \epsilon X$ we have

$$\lim n \to\infty\ T^n(x) = x_0$$

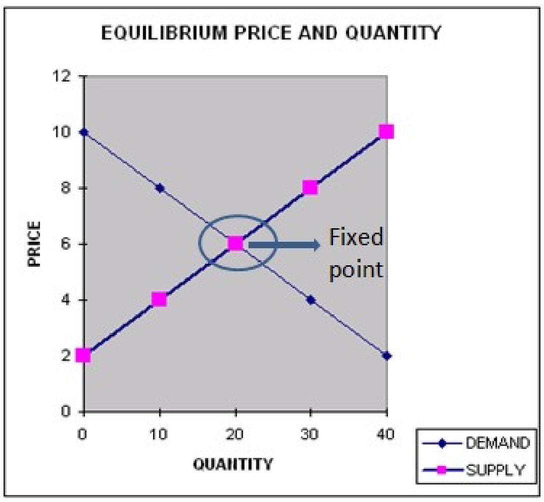
Moreover for each x , we have

$$d(T^n(x),(x_0) \leq \frac{(\sigma(T))^n}{1-\sigma(T)} d(T(x),x)$$

Market Equilibrium **:** Market equilibrium is a market state where the supply in the **market** is equal to the demand in the market. The equilibrium price is the price of a good or service when the supply of it is equal to the demand for it in the market

Fixed Point in Market Equilibrium

When the supply and demand curves intersect, the market is in equilibrium or fixed point in market . This is where the quantity demanded and quantity supplied is equal. The corresponding price is the equilibrium price or market-clearing price, the quantity is the equilibrium quantity.



Putting the supply and demand curves from the previous sections together. These two curves will intersect at Price = $6, and Quantity = 20.

In this market, the equilibrium price is $6 per unit, and equilibrium quantity is 20 units.

At this price level, market is in equilibrium. Quantity supplied is equal to quantity demanded (Qs = Qd).

Market is clear.

Fredholm Integral Equation

The most basic type of integral equation is called a Fredholm Integration *of the first type*,

$$f(x) = \int_a^b K(x,y)u(y)dy \qquad f = Ku \quad \text{non-homogeneous equation}$$

If $f(x) = 0$ then $\int_a^b K(x,y)u(y)dy = 0$ $\qquad Ku = 0$ homogeneous equation

Here u is an unknown function, f is a known function, and K is another known function of two variables, often called the kernel function. Note that the limits of integration are constant: this is what characterizes a Fredholm equation by [1]-[7],[13]

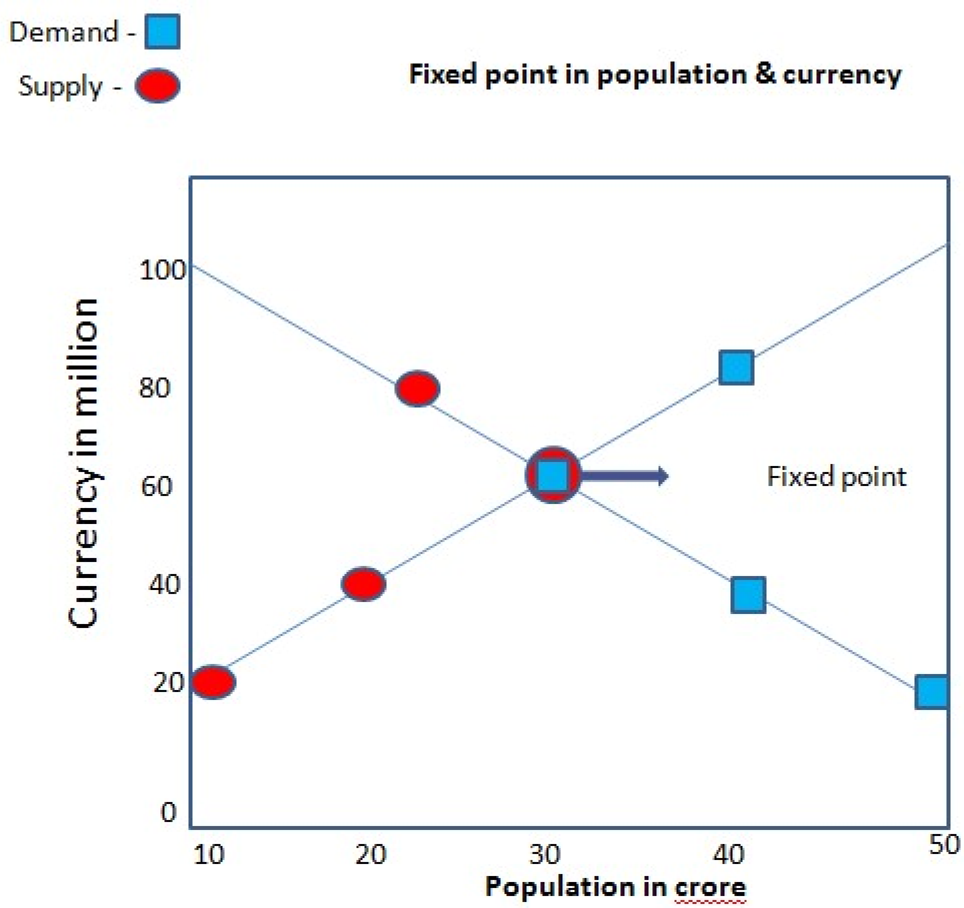
If the unknown function occurs both inside and outside of the integral, the equation is known as a *Fredholm equation of the second type*,

$u(x) = f(x) + \lambda \int_a^b k(x,y)u(y)dy$ $\quad u = \lambda Ku + f$ non-homogeneous equation

$u(x) = \lambda \int_a^b k(x,y)u(y)dy$ $\quad u = \lambda Ku$ homogeneous equation

Main Results

Fixed point theory in Population & Currency



At a point when Supply & demand curve intersect is called Fixed Point.

If the population is 30 crore, and equilibrium currency quantity is 60 million, that point is supply and demand curves intersect is called Fixed Point between the population & currency verses curve.

$$\text{Currency } \alpha \ \frac{Supply}{Demand}$$

Supply $= S_1, Demand = D_1, Currency = C_0$

Currency is directly proportional to supply and inversely proportional to demand .If demand of currency is increased than surely its supply is also increases. At a point in which supply & demand curve intersect is called fixed point .

Formula Used **:** Fredholm Integration *of the Second type* by [8]-[12].

$$u(x) = \int_a^b K(x,y)u(y)dy + f(x) \qquad u = \lambda Ku + f$$ non-homogeneous equation

$$\int_a^b K(x,y)u(y)dy = 0 \qquad Ku = 0$$ homogeneous equation

Calculation

Existence of the solution of Fredholm Integral equation on Currency function

Suppose that currency is a function of supply & demand . We denote a $C_0(S_1, D_1)$.

Consider a Fredholm's integral equation of II type

$$u = \lambda Ku + f \qquad \text{(i)}$$

Appling in currency function $C_0 = \lambda F C_0 + f$

With bounded integral operator K which also satisfies lipschitz condition

$$\| KS_1 - KD_1 \| = k \ \| S_1 - D_1 \| \ , \qquad k \geq 0$$

Rewrite integral equation in the form

$$C_0 = TC_0 \quad \text{(by def.of Fixed point)} \qquad \text{(ii)}$$

Where operator T is defined by

$$TC_0 = \lambda KC_0 + f \quad \therefore operator\ T\ is\ not\ linear$$

If C_0 is a fixed point of operator equation ,then C_0 is a solution of Integral equation.

Consider

$$\| TS_1 - TD_1 \| = \| \lambda KS_1 + f - (\lambda KD_1 + f) \|$$

$$= \| \lambda KS_1 - \lambda KD_1 \|$$

$$= |\lambda| \ \| K(S_1 - D_1) \|$$

$$\leq |\lambda| k \parallel S_1 - D_1 \parallel \qquad \therefore S_1 = D_1$$

If $|\lambda| k < 1$,then operator T is contraction and according to Banach fixed point theorem , there exist a unique fixed point of equation (ii). This unique fixed point is also a solution of Frdholm's equation.

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