Sensitivity Analysis of Inventory Management Systems without Shortages in Fuzzy Environment using Random Trapezoidal Fuzzy Numbers

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Abstract- Sound knowledge in inventory levels are essential to ensure the business companies to plan and run efficiently with optimal finances. Keeping accurate track of inventory always helps to identify data where the manufacturer plans in providing a Quality of Service(QoS) to the customers in knowing what is the inventory, and when to order for the new inventory. In this paper, Inventory models without shortages in a fuzzy environment is considered, determined the Economic Ordered Quantity(EOQ) and the total optimal cost by using Random Trapezoidal fuzzy numbers. Signed distance method is used for deffuzzification and the sensitivity analysis is studied. The proposed method of generating random fuzzy numbers and comparing these methods shows the non-linearity with the parameters in inventory.

Keywords–Economic Ordered Quantity(EOQ), deffuzzification, sensitivity analysis, Random Trapezoidal fuzzy numbers.

I. INTRODUCTION

Inventory management plays a vital role in supply chain which includes production, procurement, fulfillment, as the company has to balance customers demand with the storage space and financial issues. The expansion of inventory management control models with their use are motivated and extended by industries as well also by academicians[1, 2, 3]. With such an significance, the analysis in finding EOQ/EPQ has grown, but there are several weakness and one of obvious reason are with the number of unrealistic/impracticable assumptions[4]. The consideration of roughness and fuzziness with uncertainty plays an important role in research and our study investigates on these phenomena for EOQ/EPQ model by considering the quantity without shortage with the sensitivity analysis[5]. In many situations, the uncertainties are due to fuzziness and in fuzzy set theory these cases in detail were established by Zadeh[6]. Gupta and Kauffmann provided with an introduction of fuzzy arithmetic operations and Zinmerman gave discussions on the concepts of fuzzy set theory with its applications[7,8]. As fuzziness is the nearest and closet possible approach to the facts and reality, the fuzzy set theory in inventory modeling provides an validity to the model formulated[9]. Deterioration can be defined as damage or decay, and such an item cannot be used for its controlled reasons and almost every physical good certainly undergo deterioration or decay over a period of time. Shortages in inventory management systems are either completely totally lost or backlogged and the backlogging rate time dependent [10]. In an inventory management system, the holding cost, shortages cost, and setup cost are commonly expressed in general terms as such setup cost is likely between 15% and 35% of the unit price and is very practical to differentiate these costs in terms of the fuzzy variables11]. Rawat and De P K, proposed EOQ/EPQ model without shortages using triangular fuzzy number and used signed distance method to compute the total cost[12]. Dutta and Pavan Kumar, proposed an inventory model by considering the setup cost and the carrying cost as trapezoidal fuzzy number and for defuzzification they used signed distance method and computed the total cost[13].

In this paper, we developed EOQ model without shortages model by considering carrying cost and setup cost with the randomly generated trapezoidal fuzzy number and the signed distance method is used for defuzzification and compute the total cost. In Section I, Introduction and literature survey is presented. Section II presents, Definitions, Preliminaries and related algorithms are given. In section III, optimal solution and sensitivity analysis for a numerical example is given. Conclusions are given in section IV.

II. DEFINITIONS AND PRELIMANRIES

2.1 Fuzzy Set, $\alpha -$ cut of a Set:

A fuzzy set, $\overline{A}$ on universal set X is the set of ordered pairs $\widetilde{A} = \{(X, \mu_A(x)) : x \in X\}$

… (1)

Here $\mu_A : X \to [0,1]$, is called the membership function of $\overline{A}$.

The $\alpha -$ cut of the fuzzy set $\overline{A}$, is defined as $A_\alpha = \{x : \mu_A(x) = \alpha, \alpha \geq 0\}$

… (2)

2.2 Properties of a Membership Function of a Set:

On the real line R, a fuzzy number then is defined as a fuzzy set $\overline{A}$ with its membership function $\mu_A : X \to [0,1]$, under the following properties:

i. $\overline{A}$ is piece-wise continuous.
ii. $\overline{A}$ is normal, that is there exist $x \in X$ such that $\mu_A(x) = 1$.
iii. $\overline{A}$ is a convex fuzzy set.
iv. $Sup(\bar{A}) = cl\{x \in R : \mu_A(x) > 0\}$, where cl represents the closure of a set.

2.3 Trapezoidal Fuzzy Number:

A trapezoidal fuzzy number $\bar{A} = (a_1, a_2, a_3, a_4)$ with its membership function $\mu_{\tilde{A}}$ is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ R(x) = \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4 \\ 0, & otherwise \end{cases}$$

… (3)

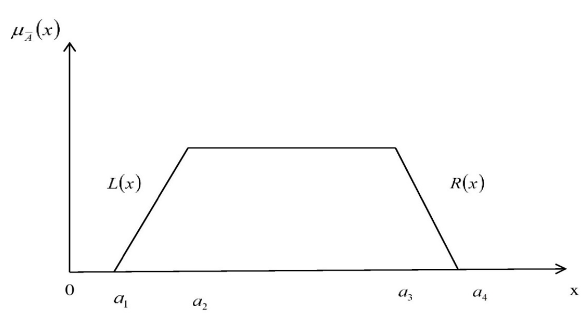

Figure 1 : Diagramatic reresentation of a Trapezoidal Fuzzy Number

2.4 LR Form of a Fuzzy Number:

An LR -Form of a fuzzy set, if exists, then for reference functions $L(x)$, $R(x)$, there exists scalars $m > 0$ and $n > 0$ with its membership function as

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{\sigma - x}{m}\right), & x \le \sigma; \\ 1, & \sigma \le x \le \gamma; \\ R\left(\frac{x - \gamma}{n}\right), & x \ge \gamma; \end{cases} \quad \dots (4)$$

Where σ, is the mean value of $\bar{A}$. The functions $L(x)$ and $R(x)$ map from $R^{+} \to [0,1]$, and are decreasing. A LR -Form of fuzzy numbers can be represented as $\bar{A} = (\sigma, \gamma, m, n)_{LR}$.

2.4 Operations on Fuzzy Number:

Suppose $\bar{A} = (a_1, a_2, a_3, a_4)$ and $\bar{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, then arithmetical operations are defined as:

1. $\bar{A} \oplus \bar{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
2. $\bar{A} \otimes \bar{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$
3. $\bar{A} - \bar{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
4. $\bar{A} / \bar{B} = (\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1})$
5. $\alpha \otimes \bar{A} = \begin{cases} (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4), \alpha \ge 0 \\ (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4), \alpha < 0 \end{cases}$

2.4 Signed Distance of $\bar{A}$:

$\bar{A}$ be a fuzzy set defined on R. Then the signed distance of $\bar{A}$ is defined as:

$$d(\tilde{A},0) = \frac{1}{2} \int [A_L(\alpha) + A_R(\alpha)] d\alpha \quad \dots (3)$$

Where, $\alpha -$ cut of fuzzy set $\bar{A}$ is $A_\alpha = [A_L(\alpha), A_R(\alpha)] = [a_1 + (b_1 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha], \alpha \in [0,1]$ … (4)

2.4 Physical Interpretation of Signed Distance of $\bar{A}$:

The signed distance $d(a,0) = a$, for all $a, 0 \in R$.

1. If $0 < a$, then the distance between a and 0 is $d(a,0) = a$.
2. If $a < 0$ then the distance between a and 0 is $-d(a,0) = -a$.

III: BASIC ASSUMPTIONS AND NOTATIONS

3.1 Notations:

We use the following symbols:

c_1	:	carrying/holding cost per unit quantity per unit time
c_3	:	setup/ordering cost per order
T	:	length of the plan
q^*	:	ordered quantity per cycle
n	:	length of one cycle
R	:	total demand during the time period [0,T]
TC	:	total cost for the period [0, T]
$T\overline{C}$	:	fuzzy total cost over the period [0, T]
$F(T_q)$	:	defuzzified total cost for [0, T]
$F(T_{q^*})$	:	minimum defuzzified total cost for [0, T]
EOQ	:	optimal ordered quantity

3.2 Assumptions:

The following are the assumptions in finding optimal quantity :

a) Shortages are not allowed.
b) Total demand is considered as constant.
c) Setup cost and holding cost are fuzzy in nature.
d) Time/Duration is constant.

IV : INVENTORY MODEL IN CRISP NATURE

Here we, first considered an inventory management model without shortages, in crisp nature and then the economic ordered size is obtained by the modeled equation as:

$$q^* = \sqrt{\frac{2c_3R}{c_1T}}, \text{ where } \frac{q^*}{n} = \frac{R}{T} \quad \ldots (5)$$

The total cost over the period [0, T] is equal to the sum of the carrying cost and the ordering cost.

$$TC = \frac{c_1T_q}{2} + \frac{c_3R}{q^*} \quad \ldots (6)$$

The optimum EOQ and TC^* is then obtained by equating the first order partial derivatives of TC to zero, the Optimal ordered quantity , and Minimum total cost are obtained by the equations as:

$$\text{Optimal ordered quantity } EOQ = \sqrt{\frac{2c_3R}{c_1T}} \quad \ldots (7)$$

$$\text{Minimum total cost } TC^* = \sqrt{2c_3c_1RT} \quad \ldots (8)$$

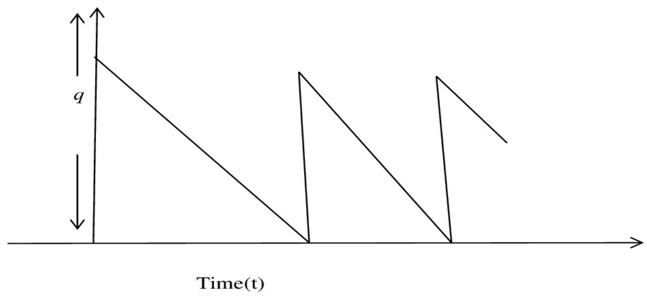

Figure 2 : Variation in q^* over time t

V: PROPOSED MODEL FOR INVENTORY MODEL IN FUZZY NATURE

The model discussed above can also be modeled in fuzzy environment. As the holding cost and the ordering cost are fuzzy in nature, we use trapezoidal fuzzy numbers to represent them.

Let $\bar{c}_1$: fuzzy holding/ carrying cost per unit quantity per unit time

$\bar{c}_3$: fuzzy set up/ordering cost per order

The time of plan and total demand are considered to be as constants. the fuzzified total cost is given by

$$T\overline{C} = \frac{\bar{c}_1 T_q}{2} + \frac{\bar{c}_3 R}{q^*} \qquad \ldots(9)$$

The Signed distance method is used in defuzzifying the fuzzy total cost, and then we obtain the optimal ordered quantity (EOQ).

Suppose $\bar{c} = (c_1, c_2, c_3, c_4)$, and $\bar{s} = (s_1, s_2, s_3, s_4)$ are fuzzy trapezoidal numbers in LR-form, where $0 < \bar{c}_3 < \bar{c}_1$ and $s_1, s_2, s_3, s_4, c_1, c_2, c_3, c_4$ are given positive numbers.

From equation (9), we have:

$$\begin{aligned} T\overline{C} = T\overline{C}(\bar{c}_1, \bar{c}_3) &= \left[\bar{c}_1 \otimes \frac{T_q}{2}\right] \oplus \left[\bar{c}_3 \otimes \frac{R}{q^*}\right] \\ &= \left[(c_1, c_2, c_3, c_4) \otimes \left(\frac{T_q}{2}\right)\right] \oplus [(s_1, s_2, s_3, s_4)] \otimes \left(\frac{R}{q^*}\right) \\ &= \left(c_1 \frac{T_q}{2}, c_2 \frac{T_q}{2}, c_3 \frac{T_q}{2}, c_4 \frac{T_q}{2}\right) \oplus \left(s_1 \frac{R}{q^*}, s_2 \frac{R}{q^*}, s_3 \frac{R}{q^*}, s_4 \frac{R}{q^*}\right) \\ &= \left(c_1 \frac{T_q}{2} + s_1 \frac{R}{q^*}, c_2 \frac{T_q}{2} + s_2 \frac{R}{q^*}, c_3 \frac{T_q}{2} + s_3 \frac{R}{q^*}, c_4 \frac{T_q}{2} + s_4 \frac{R}{q^*})\right) \\ &= (a_1, a_2, a_3, a_4) \text{ (Let)} \end{aligned} \qquad \ldots(10)$$

Now, $A_L(\alpha) = a_1 + (a_2 - a_1)\alpha = c_1 \frac{T_q}{2} + s_1 \frac{R}{q^*} + \left[(c_2 - c_1)\frac{T_q}{2} + (s_2 - s_1)\frac{R}{q^*}\right]\alpha$

And, $A_R(\alpha) = a_4 - (a_4 - a_3)\alpha = c_4 \frac{T_q}{2} + s_4 \frac{R}{q^*} + \left[(c_4 - c_3)\frac{T_q}{2} + (s_4 - s_3)\frac{R}{q^*} \right]\alpha$

Deffuzzifying $\overline{TC}$ in (10) by using the Signed distance method, we get

$$d(\overline{TC}(\overline{c}_1, \overline{c}_3), 0) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha)]d\alpha$$

$$= \frac{1}{2}\left[(c_1 + c_4)\frac{T_q}{2} + (s_1 + s_4)\frac{R}{q^*} \right] + \frac{1}{4}\left[(c_2 + c_3 - c_1 - c_4)\frac{T_q}{2} + (s_2 + s_3 - s_1 - s_4)\frac{R}{q^*} \right]$$

$$= [2(c_1 + c_4) + (c_2 + c_3 - c_1 - c_4)]\frac{T_q}{8} + [2(s_1 + s_4) + (s_2 + s_3 - s_1 - s_4)]\frac{R}{4q^*}$$

$$= [(c_1 + c_2 + c_3 + c_4)]\frac{T_q}{8} + [(s_1 + s_2 + s_3 + s_4)]\frac{R}{4q^*}$$

$$= F(T_q) \text{ (Let)} \quad \ldots(11)$$

Computation of EOQ at which $F(T_q)$ is minimum:

$F(T_q)$ is minimum when $\frac{dF(T_q)}{dq} = 0$, and where $\frac{d^2F(T_q)}{dq^2} > 0$

For $\frac{dF(T_q)}{dq} = 0$, the economic ordered quantity is given by $EOQ = \sqrt{\frac{2R(s_1 + s_2 + s_3 + s_4)}{T(c_1 + c_2 + c_3 + c_4)}}$

Also, at $q^* = EOQ$, we have $\frac{d^2F(T_q)}{dq^2} > 0$ …(12)

This shows that $F(T_q)$ is minimum at $q^* = EOQ$. From equation 11, we have

$$F(T_q) = (c_1 + c_2 + c_3 + c_4)\frac{TEOQ}{8} + (s_1 + s_2 + s_3 + s_4)\frac{R}{4EOQ} \quad \ldots(13)$$

Algorithm for Finding Fuzzy Total Cost and Fuzzy Optimal Order Quantity:

1. Calculate Total cost TC for the crisp model as given in equation 6 using $\overline{c}_1, q^*, T,$ and R .
2. Now, determine fuzzy total cost $\overline{TC}$ with fuzzy ordering cost and fuzzy carrying cost using fuzzy trapezoidal numbers.
3. Use signed distance method in finding $\overline{TC}$, and fuzzy optimal ordered quantity.

VI : NUMERICAL EXAMPLE

For a Crisp Model: Let the setup and carrying cost for the commodity be $c_3 = Rs.20/-$ and $c_1 = Rs.12/-$ per unit respectively. Let the demand be $R = 500$ per units per day and the length of the plan be $T = 6$ days. The economic ordered quantity and total cost are respectively, $q^* = 16.67$ units, $TC = 1200$.

For a Fuzzy Model: Let the setup and carrying cost for the commodity be $c_3 = (15,19,21,24)$ and $c_1 = (8,11,13,16)$ respectively. Let the demand be $R = 500$ per units per day and the length of the plan be $T = 6$ days. The economic ordered quantity and total cost are respectively, $q^* = 16.56$ units, $F(q)^* = Rs.1192.47/-$

For Sensitivity Analysis: Let $c_3 = (10,17,18,20)$, $c_1 = (11,14,18,20)$, Demand $R = 500$ and $T = 6$ days. Then, we get the same value of $q^* = 13.11$ units, $F(q)^* = Rs.1239.2/-$

S.NO	Demand(R)	$c_3 = (15,19,21,24)$ $c_1 = (8,11,13,16)$		$c_3 = (10,17,18,20)$ $c_1 = (11,14,18,20)$	
		$F(q)^*$	q_d^*	$F(q)^*$	q_d^*
1	450	1131.28	15.71	1175.61	12.44
2	475	1162.24	16.14	1207.82	12.78
3	**500**	**1192.47**	**16.56**	**1239.2**	**13.11**
4	525	1221.92	16.97	1269.8	13.43
5	550	1250.67	17.37	1299.68	13.75

Table 1: Sensitivity Analysis for Economic Ordered Quantity and Total Cost

VII.CONCLUSIONS

Signed distance method is used to defuzzify the holding cost and setup cost. Defuzzification is done by using trapezoidal fuzzy numbers and Economic Ordered Quantity and Total Cost is calculated for both crisp and fuzzy quantities, The changes are observed. Here random trapezoidal fuzzy numbers are taken into consideration and comparison was done the existing results. Sensitivity analysis was made, and a small variation in the associated costs, there was a change in the EOQ and Total Cost.. It is observed that the EOQ is decreased with the existing results, whereas the total cost is increased. From this results using random numbers, better predictions and conclusions can be made in maintaining the inventory management.

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