Application of Differential Quadrature Method in Runge- Kutta Method

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Abstract - In this research paper we can calculate the accuracy of slope in Runge-Kutta method by the employ of and differential Quadrature methodwith a linear and nonlinear behavior of differential problem.This method is based on the Runge-Kutta method on order four .This result is based on numerical problem of ordinary differential equation of initial and Boundary value problem.

Keywords: Runge-Method, Differential Quadrature, Initial and boundary value problem, Ordinary differential equations.

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I. INTRODUCTION

In 1970 Bellman proposed the differential quadrature method (DQM) and his associateswith [1-3] efficient numerical method to solve non-linear partial differential equations. The DQM has been useful for many areas in engineering and sciences and is gradually emerging as a distinct numerical solution technique and ability to yield highly accurate approximations. Thismethod is also compared to finite difference and finite element method. Advantages of DQM method is analogs of differential equations with variable coefficients can be implemented directly.The DQM have developed in form of generalized collocation method orcollocation-interpolation method [4, 5].The study of review paper [5] provides the survey of updating literature of DQM's applications. In Conventional DQM[5] study the differential equations of order two or more than at one condition of each discrete point. In structural mechanics is also governed by high-order differential of order .

For example, plate of thin beam is fourth order differential equation and is constrained by two boundary condition at each boundary .In DQM the functional value of independent variable at any point,and its each edge is implementation of two boundary condition. If domain is discretized in normal way then the number of resulting differential quadrature with analog equations will be more than the number of functional values at that point of resulting equations cannot be solved. In boundary condition of fourth order system at $\delta -$ point approximation in sampling poin was first proposed by Jang [6] at small distance $\delta \cong 10^{-5}$(in dimensional less value) adjacent to the boundary point is introduced by Gutierrez and Laura[7] , applied to this $\delta - point$ technique to a sixth order differential equation of structural ring which is constrained by three boundary condition at each point . This technique to be applied the solution of governing equation at more than one point and choose the successive $\delta\ points.$

Bert and Malik [8] given the first extensive application of DQM in high order structural mechanics and multiple boundary value problems and its application is also define in beam ,plate and shell structures, which indicate the δ point technique is one representative step in the development of DQM. Mansell [9] introduce the orthogonal

polynomial as test functions to construct the weighting coefficient of the DQM. Chang[10] is utilized the orthogonal function as test functions ,had the aid of auxiliary functions to increase the accuracy of solutions. It is also employed in domain decomposition and mapping techniques of the DQM [11-16].
The author investigated the DQM it can be applied to eight order differential equation without recourse to special technique such as $\delta\ point$ technique which can be used to one equation at discrete points.

II. DIFFERENTIAL QUADRATURE

Suppose a function $\psi(x)$ is define a differential equation in any domain . The function values of finite set of N discrete points $x_i = (i = 1,2,\ldots\ldots N)$.
Let $\psi_i = \varphi_i(x_i)$is denote the function values. The DQM express the derivative at discrete points x_i.

$$\psi^r(x_i) = \frac{d^r\varphi(x_{i)}}{dx^r} i = 1,2,\ldots\ldots.N, r \geq 1$$

In Differential Quadrature various type of test function have been used, including the polynomials, sine and cosine functions [17], the Lagrange interpolated-based trigonometric polynomials [18, 19], sine function [5], spline function and various orthogonal polynomials [9, 10] such as Jacobi, Laguerre, Hermite and Chebyshev polynomials.
In this research paper we calculate the accuracy of slope in Runge-kutta method at discrete point by employing the DQM method.

III. RUNGE'SMETHOD

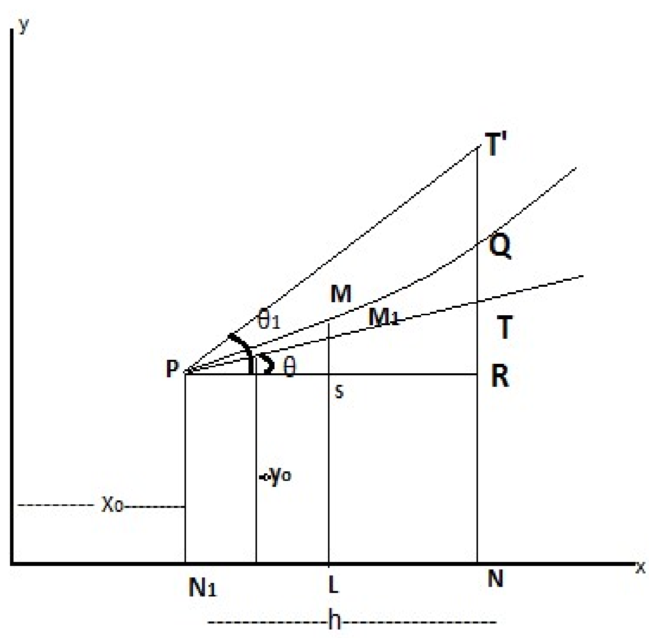

Fig 1

IV. RUNGE-KUTTA METHOD OF FOURTH ORDER

Runge-Kutta method is one of the most widely used methods. This method does not involve the derivatives of $f(x,y)$. There is a set of methods known as Runge-Kutta methods known as Runge-Kutta methods .But the most widely used method is Runge-Kutta of fourth order and so this method is generally given the name Runge-kutta method.

Consider the differential equation of first order

$$y' = \frac{dy}{dx} = f(x,y)$$

with the initial condition $y = y_0$ when $x = x_0$.

Let $[x_0, x_0 + h]$ be the first subinterval, By this method we can calculate the increment in the first sub-interval as under:

$K_1 = hf(x_0, y_0) = h \times$ Slope at the beginning of the first sub-interval,

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{1}{2}K_1\right)$$

$$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{1}{2}K_2\right)$$

$K_4 = hf(x_0 + h, y_0 + K_3) = h \times$ Slope at the end of the first sub-interval

In compare to DQM

$$y' = \frac{dy}{dx} = f(x,y)$$

$$\frac{d^r\varphi(x_i)}{dx^r}$$

$i = 1,2,\ldots\ldots.N, r \geq 1$ is discrete point , In runge-kutta method this point is define on sub interval of slope K_1, K_2, K_3 and so on …..

Example 1. Solve DQM by the use of Runge-Kutta method at $y(0.2)$ for the equation $\frac{d^2y}{dx^2} = x\frac{dy}{dx} - y$.

Initial condition are $x = 0, y = 1, \frac{dy}{dx} = 0$.

Solution .Putting $\frac{dy}{dx} = z = f(x,y,z)$

The given equations becomes $\frac{dz}{dx} = xz - y = \phi(x,y,z)$

The initial conditions are $x = 0, y = 1, z = 0$

$x_0 = 0, y_0 = 1, z_0 = 0$ and $h = 0.2.$

Using S_1, S_2, S_3, S_4 for $f(x,y,z)$ and k_1, k_2, k_3, k_4 for $\phi(x,y,z)$, we have

$$S_1 = hf(x_0, y_0, z_0) = hz_0 = (0.2) \times 0 = 0$$

$$K_1 = h\phi(x_0, y_0, z_0) = h(x_0 z_0 - y_0)$$
$$= 0.2\ (0 \times 0 - 1) = -0.2$$
$$S_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{1}{2}S_1, z_0 + \frac{1}{2}k_1\right)$$
$$(0.2)\left(z_0 + \frac{1}{2}k_1\right) = (0.2)[0 + (-0.1)] = -0.02$$
$$k_2 = h\phi\left(x_0 + \frac{h}{2}, y_0 + \frac{1}{2}S_1, z_0 + \frac{1}{2}k_1\right)$$
$$h\left\{\left(x_0 + \frac{1}{2}h\right)\left(z_0 + \frac{1}{2}k_1\right) - \left(y_0 + \frac{1}{2}S_1\right)\right\}$$
$$= 0.2\ \{(0 + 0.1)(0 - 0.1) - (1 + 0)\} = -0.2020$$
$$S_3 = hf\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}S_2, z_0 + \frac{1}{2}k_2\right),$$
$$h\left\{z_0 + \frac{1}{2}k_2\right\} = 0.2(0 - 0.1010) = -0.202$$
$$k_3 = h\phi\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}S_2, z_0 + \frac{1}{2}k_2\right)$$
$$h\left\{\left(x_0 + \frac{1}{2}h\right)\left(z_0 + \frac{1}{2}k_2\right) - \left(y_0 + \frac{1}{2}S_2\right)\right\}$$
$$0.2\{(0.1)(-0.1010) - (1 - 0.01)\} = -0.20002$$

$$S_4 = hf(x_0 + h, y_0 + S_3, z_0 + k_3)$$
$$= (0.2)(-0.20002) = -0.040004$$
$$k_4 = h\phi(x_0 + h, y_0 + S_3, z_0 + k_3)$$
$$= h\{(x_0 + h)(z_0 + k_3) - (y_0 + S_3)\}$$
$$= 0.2\ \{(0.2)(-0.20002) - (1 - 0.0202)\} = -0.2039608.$$
$$x = x_0 + h = 0 + 0.2 = 0.2$$
$$\therefore \quad y(0.2) = y_0 + \frac{1}{6}(S_1 + 2S_2 + 2S_3 + S_4)$$

$$= 1 + \frac{1}{6}\{0 + 2(-0.02) + 2(-0.0202) - (0.040004)\} = 0.97993267$$

$$z(0.2) = z_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = -0.20133347$$

Result: By the use of RungeKutta method of fourth order we calculate the DQM at discrete point of slope K_1, K_2, K_3, K_4 ... if initial condition are given.

REFERENCES

[1] R. BELLMAN and J. CASTI 1971 Journal of Mathematical Analysis and Applications 34, 235-238.Differential quadrature and long term integration.

[2] R. BELLMAN, B. G. KASHEF and J. CASTI 1972 Journal of Computational Physics 10, 40}52. Di!erential quadrature: a technique for the rapid solution of non-linear partial di!erential equations.

[3] R. BELLMAN, B. KASHEF and R. VASUDEVAN 1974 Mathematical Bioscience 19, 221-230. The inverse problemof estimating heart parameters from cardiograms.

[4] B. A.FINLAYSON 1980 Nonlinear Analysis in Chemical Engineering. New York: McGraw-Hill.

[5] N. BELLOMO 1997 Mathematicaland Computer Modelling26, 13}34. Nonlinear models and problems in applied sciences from differential quadrature to generalized collocation methods.

[6] S. K. JANG, C. W. BERT and A. G. STRIZ 1989 International Journal for Numerical Methods in Engineering 28, 561-577. Application of differential quadrature to static analysis of structural components.

[7] R. H. GUTIERREZ and P. A. A. LAURA 1995 Journal of Sound and vibration 185, 507-513.Vibrations of non-uniform rings studied by means of the differential quadrature method.

[8] C. W. BERT andM. MALIK 1996 Applied Mechanics Review 49, 1}27. Differential quadrature method in computational mechanics: a review.

[9] G. MANSELL, W. MERRYFIELD and B. SHIZGAL 1993 Computer Methods in Applied Mechanics and Engineering 104, 295}316. A comparison of differential quadrature methods for the solution of partial differential equations.

[10] C. T. CHANG, C. S. TSAI and T. T. LIN 1993 Chemical Engineering Communications 123, 135-164.The modified differential quadrature and their applications.

[11] F. CIVAN and C. M. SLIEPCEVICH 1985 Proceedings of Oklahoma Academic Sciences 65, 73-79.Application of differential quadrature to solution of pool boiling in cavities.

[12] S. S. E. LAM 1993 Computers and Structures 47, 459-464. Application of the differential quadrature method to two-dimensional problems with arbitrary geometry.

[13] C. W. BERT and M. MALIK 1996 International Journal of Mechanical Sciences 38, 589-606. The differential quadrature method for irregular domains and application to plate vibration.

[14] K. M. LIEW and J.-B. HAN 1997 Communications in Numerical Methods in Engineering 13, 73}81. A four-node differential quadrature method for straight-sided quadrilateral Reissner/Mindlin plates.

[15] J.-B. HAN and K. M. LIEW 1997 Computer Methods in Applied Mechanics and Engineering 141, 265-280. An eight-node curvilinear differential quadrature formulation for Reissner/Mindlin plates.

[16] H. ZHONG and Y. HE 1998 International Journal of Solids and Structures 35, 2805-2819. Solution of Poisson and Laplace equations by quadrilateral quadrature element. vibration analysis of circular plates with stepped thickness over a concentric region by the di!erential quadrature element method.

[17] A. G. STRIZ, X. WANG andC. W. BERT 1995 AC¹A Mechanica 111, 85}94. Harmonic differential quadrature method and applications to analysis of structural components.

[18] C. SHU and Y. T. CHEW 1997 International Journal for Numerical Methods in Engineering 13,643-653. Fourier expansion-based differential quadrature and its applicationto Helmholtz eigenvalue problems.

[19] C. SHU and B. E. RICHARDS 1992 International Journal ofNumerical Methods in Fluids 15,791-798. Application of generalized differential quadrature to solve two-dimensional incompressibleNavierStokes equations.