Color class Dominating sets in Triangular ladder and Mobius ladder Graphs

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Abstract - Let $G = (V, E)$ be a graph. A color class dominating set of G is a proper coloring $\mathcal{C}$ of G with the extra property that every color class in $\mathcal{C}$ is dominated by a vertex in G. A colorclass dominating set is said to be a minimal color class dominating set if no proper subset of $\mathcal{C}$ is a color class dominating set of G. The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of G and is denoted by $\gamma_{\chi}(G)$. Here we obtain $\gamma_{\chi}(G)$ for Triangular ladder graph and Mobius ladder graph.
Key words: Chromatic number, Domination number, Color class dominating set, Color class domination number.
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I. INTRODUCTION

All graphs considered in this paper are finite, undirected graphs and we follow standard definitions of graph theory as found in [2].

Let $G = (V, E)$ be a graph of order p. The open neighborhood $N(v)$ of a vertex $v \in V(G)$ consists of the set of all vertices adjacent to v. The closed neighborhood of v is $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood $N(S)$ is defined to be $\bigcup_{v \in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$. For any set H of vertices of G, the induced sub graph $\langle H \rangle$ is the maximal subgraph of G with vertex set H.

A subset S of V is called a dominating set if every vertex in $V - S$ is adjacent to some vertex in S. A dominating set is a minimal dominating set if no proper subset of S is a dominating set of G. The domination number $\gamma(G)$ is the minimum cardinality taken over all minimal dominating sets of G. A γ −set is any minimal dominating set with cardinality γ. A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The smallest number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by $\chi(G)$.

A color class dominating set of G is a proper coloring $\mathcal{C}$ of G with the extra property that every color class in $\mathcal{C}$ is dominated by a vertex in G. A color class dominating set is said to be a minimal color class dominating set if no proper subset of $\mathcal{C}$ is a color class dominating set of G. The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of G and is denoted by $\gamma_{\chi}(G)$.This concept was introduced by A. Vijayalekshmi and A.E. Prabhain [1]

For any two graphs G and H, we define the cartesian product, denoted by $G \times H$, to be the graph with vertex set $V(G) \times V(H)$ and edges between two vertices (u_1, v_1) and (u_2, v_2) iff either $u_1 = u_2$ and $v_1 v_2 \in E(H)$ or $u_1 u_2 \in E(G)$ and $v_1 = v_2$. A ladder graph can be defined as $P_2 \times P_n$, where $n \geq 2$ and is denoted by L_n.A triangular ladder graph TL_n, $n \geq 2$ is a graph obtained from L_n by adding the edges $u_i v_{i+1}$, $1 \leq i \leq n-1$.The vertices of L_n are $u_i$and v_i ,u_i and v_i are two paths in the graph $L_n$where $i - \{1,2, \ldots n\}$. A mobius ladder graph M_n is a graph obtained from the ladder graph $P_n \times P_2$ by joining the opposite end points of the two copies of p_n.

II. MAIN RESULTS

Definition 2.1. Let G be a graph. A color class dominating set of G is a proper coloring C of G with the extra property that every color classes in C is dominated by a vertex in G. A color class dominating set is said to be a minimal color class dominating set if no proper subset of C is a color class dominating set of G. The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of G and is denoted by$\gamma_\chi(G)$.

Theorem 2.2. For the triangular ladder graph TL_n , $n \geq 2$,

$$\gamma_\chi(TL_n) = \begin{cases} n+1 & if n = 2,5 \\ n \forall n \geq 3 \; and n \neq 5 \end{cases}$$

Proof: Let $V(TL_n) = \{u_i , v_i \,/ 1 \leq i \leq n\}$ and

$E(TL_n) = \{u_i u_{i+1} \,/ i < n\} \cup \{v_i v_{i+1} \,/ i < n\} \cup \{u_i v_{i+1} \,/ i < n\}$.

When ≤ 5 , the proof is obvious.

Let $n \geq 6$ we have 4 cases.

Case(1). When $n \equiv 0 \ (mod 4)$

Decompose TL_n into $\frac{n}{4}$ copies of TL_4 and $\gamma_\chi(TL_4) = 4$.So$\gamma_\chi(TL_n) = n$

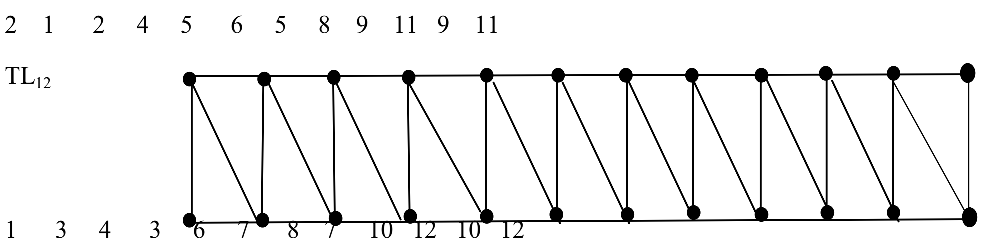

Figure 1 $\gamma_\chi(TL_{12}) = 12.$

Case (2). When $n = 1 \ (mod 4)$. We consider 3 subcases.

Subcase 2.1 When $n \equiv 0 \ (mod 3)$. In this case, decompose TL_n into $\frac{n}{3}$ copies of TL_3 and $\gamma_\chi(TL_3) = 3$.So$\gamma_\chi(TL_n) = n$.

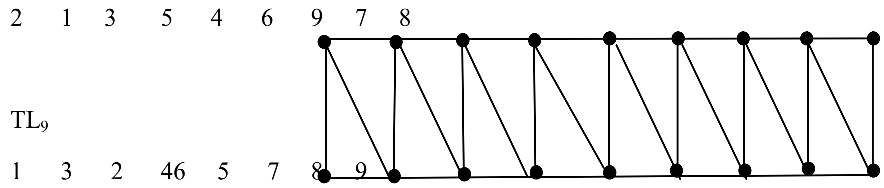

Figure 2.1 $\gamma_\chi(TL_9) = 9.$

Subcase 2.2 When $n \equiv 1(mod3)$. TL_ncan be obtained from TL_{n-4} followed byTL_4 and since $n - 4 \equiv 0\ (mod3)$. So $\gamma_\chi(TL_n) - \gamma_\chi(TL_{n-4}) + \gamma_\chi(TL_4)$. By subcase 2.1 $\gamma_\chi(TL_n) = n.$

TL_{13}

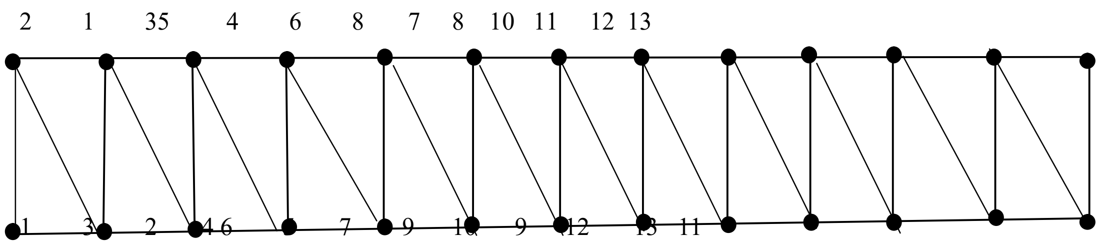

Figure 2.2 $\gamma_\chi(TL_{13}) = 13$

Subcase 2.3 When $n \equiv 2(mod3)$. Since $n - 4 \equiv 1\ (mod4)$ and $(n-4) \equiv 1\ (mod3)$, by subcase 2.2,$\gamma_\chi(TL_n) = \gamma_\chi(TL_{n-4}) + \gamma_\chi(TL_4) = n$.

TL_{17}

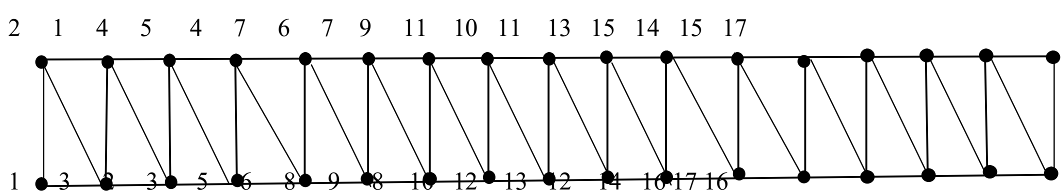

Figure 2.3 $\gamma_\chi(TL_{17})$ =17

Case (3). When$n \equiv 2(mod4)$. Here also we consider 3 subcases.

Subcase 3.1 When $n \equiv 0(mod3)$. By Subcase 2.1,$\gamma_\chi(TL_n) = n.$

TL_{18}

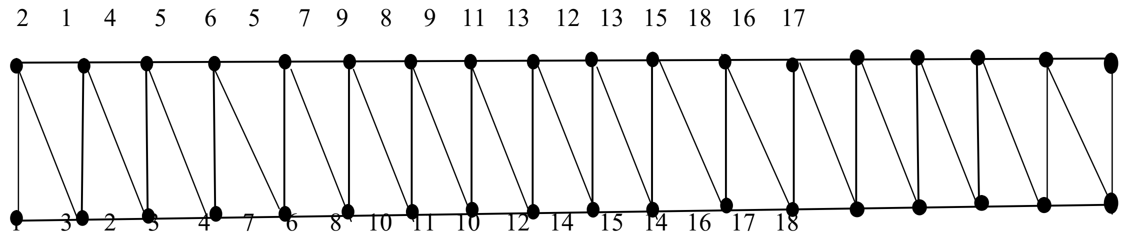

Figure 3.1 $\gamma_\chi(TL_{18}) = 18$

Subcase 3.2 When$n \equiv 1(mod3)$, TL_n is obtained from TL_{n-4} followed by TL_4. Since $n - 4 \equiv 0\ (mod3)$ and by subcase 3.1,

$\gamma_\chi(TL_n) = \gamma_\chi(TL_{n-4}) + \gamma_\chi(TL_4) = n$.

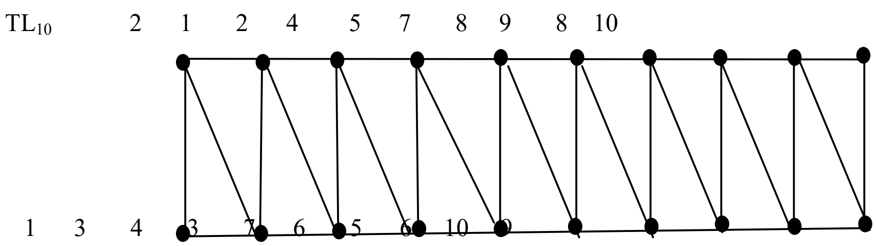

Figure 3.2 $\gamma_\chi(TL_{10}) = 10$

Subcase 3.3. When$n \equiv 2(mod3)$. TL_n is obtained from TL_{n-6} followed ByTL_6, since $n - 6 \equiv 0(mod4)$, by case (1)

$\gamma_\chi(TL_n) = \gamma_\chi(TL_{n-6}) + 2\gamma_\chi(TL_3) = n$.

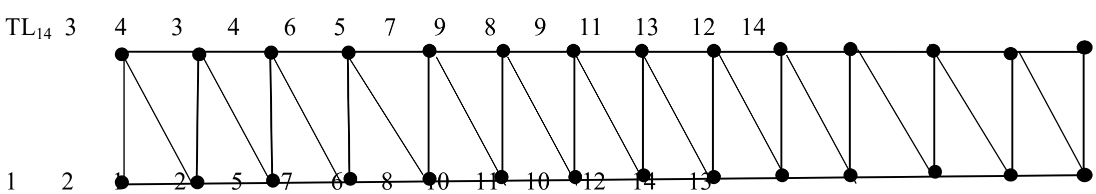

Figure 3.3 $\gamma_\chi(TL_{14}) = 14$

Case (4). When $n \equiv 3(mod4)$

Since $n - 3 \equiv 0(mod4)$ and TL_n is obtained from TL_{n-3} followed by TL_3, by case (1) $\gamma_\chi(TL_n) = \gamma_\chi(TL_{n-3}) + \gamma_\chi(TL_3) = n$.

TL_{15}

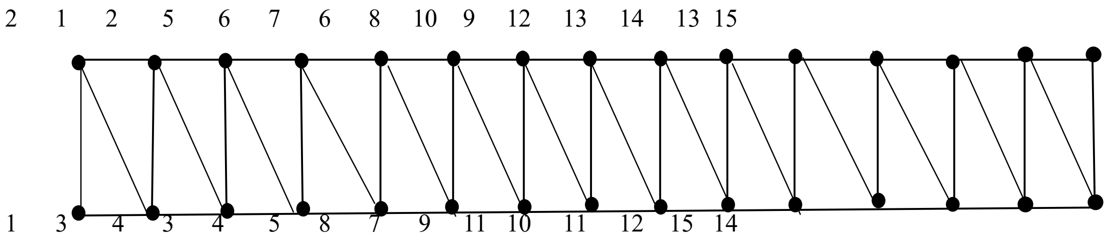

Figure 4$\gamma_\chi(TL_{15}) - 15$

Theorem 2.3. Let M_n be a mobius ladder graph.

Then $\gamma_\chi(M_n) = \begin{cases} \frac{2n}{3} \, if n = 0 \ (mod \ 3) \\ \left\lfloor \frac{2n+4}{3} \right\rfloor otherwise \end{cases}$

Proof: Let $M_n$be amobius ladder graph with

$V(M_n) = \{u_1, u_2, \dots \dots u_{n+1}, \dots . u_{2n}\}$. we take

$N(u_i) = \{u_{i-1}, u_{i+1}, u_{i+n}\}fori = 2,3, \dots . . (n-1)$

$N(u_j) = \{u_{j-1}, u_{j+1}, u_{j-n}\}forj = (n+2), (n+3) \dots \dots . (2n-1)$

$N(u_1) = \{u_2, u_{n+1}, u_{2n}\}$,

$N(u_n) = \{u_{n-1}, u_{n+1}, u_{2n}\}$

$N(u_{n+1}) = \{u_1, u_n, u_{n+2}\}$,

$N(u_{2n}) = \{u_1, u_n, u_{2n-1}\}$

We consider 3 cases

Case (1) When $n \equiv 0(mod3)$

Assign new colors, say $2i - 1, \ 2i \left(1 \le i \le \frac{n}{3}\right)$ to the vertices $N(u_i)$

$for i = 2,5, \dots \dots \dots \dots \dots (n-1)$ and$i = (n+2), (n+5) \dots \dots \dots (2n-1)$,

we get a γ_χ- coloring of M_n. So $\gamma_\chi(M_n) = \frac{2n}{3}$

2 2

1 1

M_9:2 3

5

64 3

563

5

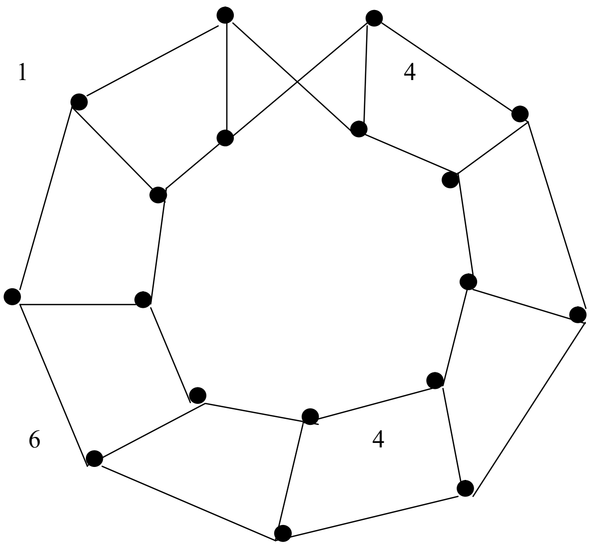

Figure 5$\gamma_\chi(M_9) = 6$.

Case (2) When $n \equiv 1(mod3)$

Since $n-1 \equiv 0(mod3)$ and by case (1),$\gamma_\chi(M_{n-1}) = \frac{2(n-1)}{3}$. Assign two new colors, say $\frac{2(n-1)}{3}$+1 and $\frac{2(n-1)}{3} + 2$ to the vertices $\{u_n\}$ and $\{u_{2n}\}$ respectively, we attain γ_χ-coloring of M_n. Thus

$\gamma_\chi(M_n) = \left\lfloor\frac{2n+4}{3}\right\rfloor$

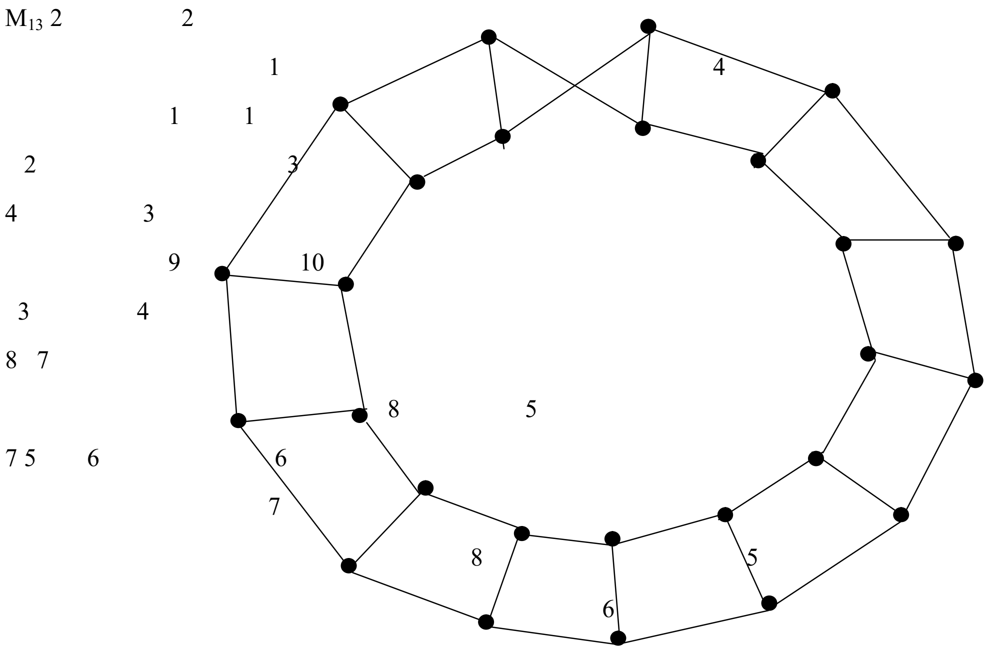

Figure 6 $\gamma_\chi(M_{13}) = 10$.

Case (3) When $n \equiv 2(mod3)$

Since $n-2 \equiv 0(mod3)$ and by case (1) $\gamma_\chi(M_{n-2}) = \frac{2(n-2)}{3}$. Assign two new color, say $(\frac{2(n-2)}{3})$+1 and$(\frac{2(n-2)}{3})$+2 to the vertices $\{u_{n-1}, u_{2n}\}$ and $\{u_n, u_{2n-1}\}$ respectively. We get a γ_χ- coloring of M_n. So $\gamma_\chi(M_n) = \left\lfloor\frac{2n+4}{3}\right\rfloor$.

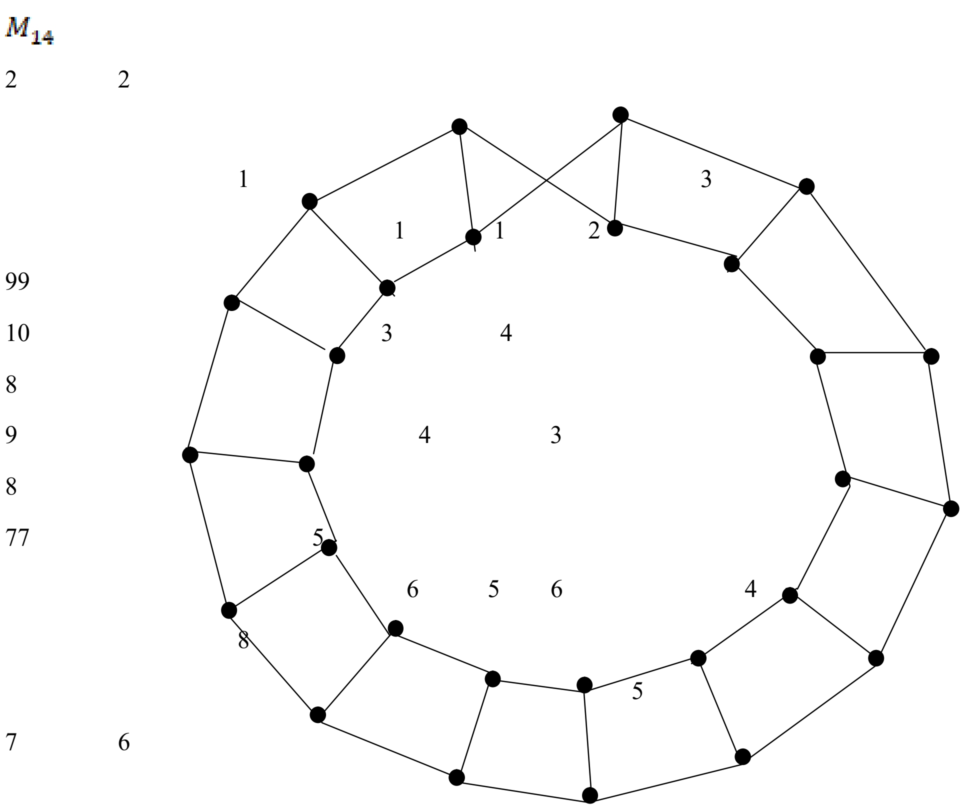

Figure 7 $\gamma_{\chi}(M_{14}) = 10$

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