"Nopassing Queue with Discouragement and Removable Servers"

Dr. Manoj Kumar Jadaun[1], Dr. Rajesh Kumar Pal[1]
[1]*Associate Professor, Department of Mathematics, D.A.V.(P.G) College, Dehradun (U.K.)*

Abstract - A multi-server markovian queueing system including the concepts of balking and reneging under the restriction of nopassing is considered. In order to reduce the discouraging behavior of the customers, there is also provision of removable servers which turn on one by one according to a threshold scheme. The customers arrive according to poisson process and leave the system in chronological order. This restriction characterizes the queueing system with two types of customers wherein the first type of customers have zero service time and second type of customers have exponentially distributed service time. We derive the expression for queue size distribution. The expected waiting time of both types of customers and difference between their expected waiting times are also derived

Keywords: markovian queueing, nopassing, poisson process.

I. INTRODUCTION

Multi-server markovian queues under the restriction of nopassing have been studied extensively by several researchers. Due to this restriction, the arriving customers may leave the system in order of their arrival. Examples of such system include a narrow boat lock (where the service time consists mostly of paper work), a remote border crossing with no parking space (wherein arrivals may be affected by the number of customers present in the system), communication network (in which packets must be transmited in a fixed order) etc.
Washburn (1) introduced the concept of nopassing for infinite capacity markovian queueing model. Sharma et al. (2) developed M/M/m queueing system for finite model with nopassing. Jain et at. (3) analyzed multi-server queueing model with discouragement and two types of customers under restriction of nopassing and obtained various performance measures. Jain et al. (4) investigated multi-server loss-delay queueing system with priority and nopasing. Jain (5) extended earlier model of infinite capacity to finite capacity and finite population models. Jain and Ghimire (6) developed M/M/m/K queue with nopassing and additional servers.

The provision of addition servers may be helpful to reduce the balking behaviour of the customers in case of longer queues which can be frequently seen in many queueing situations, such as in production process, transportation, telecommunications etc. Reynolds (7) obtained stationary solution of a multi-server queue with discouragement. Abou and Shawky (8) developed single server markiovian overflow queue with balking, reneging and additional server for longer queues. Makaddis and Zaki (9) analyzed M/M/1/∞ queueing system with additional server. Varshney et al. (10) employed the concept of additional servers for a finite space M/M/m/K model which made system more convenient to implement in practical situations. Jain (11) proposed M/M/m queue with discouragement and additional servers. Garg and Jain (12) gave the solution of $G/G^Y/m$ queueing system with discouragement via diffusion approximation. Jain and Singh (13) worked on finite priority queue with discouragement. Jain and Vaidya (14) developed finite M/M/m/K queue with discouragement by providing additional servers. Madhu Jain and Poonam Singh (15) worked on performance prediction of loss and delay Markovian queueing model with nopassing and removable additional servers**.** Abdul Rasheed, K.V. and Manoharan (16) studied a discouraged arrival Markovian queueing systems introducing self-regulatory servers and analyzed the model by deriving steady state characteristics.
We proposed a multi-server markovion queueing system with discouragement and nopassing by incorporating additional servers. The remainder of the section is organized as follows: The mathematical formulation and analysis of the model is provided in subsection 2. In subsection 3, the expression for expected waiting time for both type of customers are given Subsection 4 facilitates various performance indices. Some particular cases are discussed in subsection 5. Concluding remarks and scope of the work is highlighted in the last subsection 6.

II. MATHEMATICAL MODEL AND ANALYSIS

We consider a markovian queueing system in which customers arrive according to poisson fashion with state dependent arrival rate given by

$$\lambda_n = \begin{cases} \lambda & ;0 \le n \le s \\ \left(\dfrac{s}{n+1}\right)^{\beta} \lambda & ;s < n \le N \\ \left(\dfrac{s+j}{n+1}\right)^{\beta} \lambda & ;jN < n \le (j+1)N \\ \left(\dfrac{s+r}{n+1}\right)^{\beta} \lambda & ;rN < n \le K \end{cases} \quad (1)$$

The customers are served exponentially by s permanent and r additional servers with mean rate μ. When the number of customers is less than N, there are s servers available for service. When the number of customers is greater than N, one additional server is provided for service. In general, j (j= 1, 2, ……, r-1) servers are present for service in the system. When the number of customers is greater than rN, r additional servers are provided for service. The customers may renege exponentially with parameter α, when all servers are busy. Due to reneging, the service rate can be defined as follows:

$$\mu_n = \begin{cases} s\mu & ;0 \le n \le s \\ s\mu + (n-s)\alpha & ;s < n \le N \\ (s+j)\mu + \left(n - \overline{s+j}\right)\alpha & ;jN < n \le (j+1)N \\ (s+r)\mu + \left(n - \overline{s+r}\right)\alpha & ;rN < n \le K \end{cases} \quad (2)$$

After getting required service, the customers depart from the system in the same order in which they arrive. The queue can be considered as made up of two types of customers from service point of view. The customers having zero service time, are called of type I and the customers having exponential service times with rate μ, are called of type II. Type I customers have (1-p) proportion of the total customers and type II customers have remaining p proportion of the total customers. Now cumulative distribution function (c.d.f.) of service time for a customer can be expressed as

$F(x) = 1 - p + p\{1 - \exp(-\mu_n x)\}$; $x \ge 0$; $0 \le p < 1$. …….(3.)

Let N(t) be the number of customers of type II in the queue at time t including those who are in service but excluding those who have completed their service but are still in the system.

Denote $\rho = \lambda p / \mu < s$

Let Pn be the steady state probability that there are n customers in the system. By using product type solution, steady state probabilities Pnare given as follows:

$$
P_n=\begin{cases}
\dfrac{\rho^n}{n!}p_0 & ;0\le n\le s \\[2ex]
\dfrac{\rho^n \mu^{n-s}\left(s^{\beta}\right)^{n-s} p_0}{\left(s!\right)^{1-\beta}\left(n!\right)^{\beta}\prod\limits_{i=s+1}^{n}\left\{s\mu+\left(i-s\right)\alpha\right\}} & ;s\le n\le N \\[4ex]
\dfrac{\rho^N N^{N-s}\prod\limits_{i=1}^{j-1}\left\{\left(s+i\right)^{\beta}\lambda\right\}^{N}\left\{\left(s+j\right)^{\beta}\right\}^{n-jN} p_0}{\left(s!\right)^{1-\beta}\left(n!\right)^{\beta}\prod\limits_{i=s+1}^{N}\left\{s\mu+\left(i-s\right)\alpha\right\}\prod\limits_{l=1}^{j-1}\prod\limits_{i=lN+1}^{\left(l+1\right)N}\left\{\left(s+l\right)\mu+\left(i-\overline{s+l}\right)\alpha\right\}\prod\limits_{i=rN+1}^{n}\left\{\left(s+r\right)\mu+\left(i-\overline{s+r}\right)\alpha\right\}} & ;jN<n\le\left(j+1\right)N \\[4ex]
\dfrac{\rho^N \mu^{N-s}\left(s^{\beta}\right)^{N-s}\prod\limits_{i=1}^{r-1}\left\{\left(s+i\right)^{\beta}\lambda\right\}^{N}\left\{\left(s+r\right)^{\beta}\lambda\right\}^{n-rN} p_0}{\left(s!\right)^{1-\beta}\left(n!\right)^{\beta}\prod\limits_{i=s+1}^{N}\left\{s\mu+\left(i-s\right)\alpha\right\}\prod\limits_{l=1}^{r-1}\prod\limits_{i=lN+1}^{\left(l+1\right)N}\left\{\left(s+l\right)\mu+\left(i-\overline{s+l}\right)\alpha\right\}\prod\limits_{i=rN+1}^{n}\left\{\left(s+r\right)\mu+\left(i-\overline{s+r}\right)\alpha\right\}} & ;rN<n\le K
\end{cases}
\qquad \ldots(4)
$$

By using normalizing condition $\sum_{n=0}^{k} P_n = 1$, we obtain P_0 as

$$
P_0=\left[\begin{gathered}
\sum_{n=0}^{s}\frac{\rho^n}{n!}+\frac{1}{\left(s!\right)^{1-\beta}\left(n!\right)^{\beta}}\sum_{n=s+1}^{N}\left\{\frac{\rho^n\mu^{n-s}\left(s^{\beta}\right)^{n-s}}{\prod\limits_{i=s+1}^{n}\left\{s\mu+\left(i-s\right)\alpha\right\}}\right\} \\
+\frac{\rho^N\mu^{N-s}\left(s^{\beta}\right)^{N-s}}{\left(s!\right)^{1-\beta}\left(n!\right)^{\beta}}\sum_{jN+1}^{\left(j+1\right)}\left\{\frac{\prod\limits_{i=1}^{j-1}\left\{\left(s+i\right)^{\beta}\lambda\right\}^{N}\left\{\left(s+j\right)^{\beta}\right\}^{n-jN}}{\prod\limits_{i=s+1}^{N}\left\{s\mu+\left(i-s\right)\alpha\right\}\prod\limits_{l=1}^{j-1}\prod\limits_{i=lN+1}^{\left(l+1\right)N}\left\{\left(s+l\right)\mu+\left(i-\overline{s+l}\right)\alpha\right\}\prod\limits_{i=jN+1}^{n}\left\{\left(s+j\right)\mu+\left(i-\overline{s+j}\right)\alpha\right\}}\right\} \\
+\frac{\rho^N\mu^{N-s}\left(s^{\beta}\right)^{N-s}}{\left(s!\right)^{1-\beta}\left(n!\right)^{\beta}}\sum_{rN+1}^{k}\left\{\frac{\prod\limits_{i=1}^{r-1}\left\{\left(s+i\right)^{\beta}\lambda\right\}^{N}\left\{\left(s+r\right)^{\beta}\lambda\right\}^{n-rN}}{\prod\limits_{i=s+1}^{N}\left\{s\mu+\left(i-s\right)\alpha\right\}\prod\limits_{l=1}^{r-1}\prod\limits_{i=lN+1}^{\left(l+1\right)N}\left\{\left(s+l\right)\mu+\left(i-\overline{s+l}\right)\alpha\right\}\prod\limits_{i=rN+1}^{n}\left\{\left(s+r\right)\mu+\left(i-\overline{s+r}\right)\alpha\right\}}\right\}
\end{gathered}\right]^{-1}
\qquad .(5)
$$

III. EXPECTED WAITING TIME

Let the expected waiting times for customers of type A and type B be denoted by E(WA) and E(WB) respectively. Then expected waiting time in the steady state for a customer will be expressed as

E(W) = pE(WA) + (1-p)E(WB)

Now we have to evaluate the values of expected waiting time for type A and type B customers.

3.1 Expected Waiting Time for type A customers :

When there are n customers of type A present in the system, we obtain two cases as follows

(i) If n < s, then he will enter the service immediately, and

$$W_A = \mathrm{Max}(X_1, X_2, \ldots, X_{n+1}) \qquad \ldots(6)$$

Where X_i(i =1, 2, …., n+1) is the residual service time of the customer being served by the i^{th} server. The X_i's are independent random variable with

$$F_{x_i}(x) = 1 - \exp(-\mu x) \qquad \ldots(7)$$

Therefore,

$$\frac{F_{W_A}}{N_A}\left(\frac{x}{n}\right) = 1 - \exp(-\mu x)^{n+1} = G_{n+1}(x) \text{ for } n < s \qquad \ldots(8)$$

(ii) If n ≥ s, the waiting time of type A customers is given by

$$W_A = T + Y \qquad \ldots(9)$$

Where T is the waiting time upto entering the service and T is the time from starting of the service until leaving the system. The expectation of random variable T is given by

$$E(T) = \frac{n-k+1}{k\mu} \qquad \ldots(10)$$

And $F_Y(X) = G_k(X)$

$$k = \begin{cases} s & ; a \le n \le N \\ s+j & ; jN < n \le (j+1)N \\ s+r & ; rN < n \le K \end{cases} \qquad \ldots(11)$$

Thus, the expression for the mean waiting time of type A customer is given by

$$E\left(W_A\right)=\frac{1}{\mu}\left[\sum_{n=0}^{s}a_{n+1}+\sum_{n=s+1}^{N}\left\{\frac{n-s+1}{s}+a_s\right\}+\sum_{j=1}^{r-1}\sum_{jN+1}^{(j+1)N}\left\{\frac{n-\left(s+j\right)+1}{\left(s+j\right)}+a_{s+j}\right\}+\sum_{rN+1}^{K}\left\{\frac{n-\left(s+r\right)+1}{\left(s+r\right)}+a_{s+r}\right\}\right]P_n$$

….(12)

3.2 Expected waiting time for type B customers :

(i) **If n < s :**For the customer of type B, we obtain

$$\frac{F_{W_B}}{N_B}\left(\frac{X}{n}\right)=G_n\left(X\right) \qquad \text{…(13)}$$

(ii) If n ≥ s, in this case the waiting time W_B for type B customer is $W_B = T + Y$, where the random variable T is given by equation (6.10) and

$$F_Y(X) = G_{k-1}(X) \qquad \text{…(14)}$$

If m_n denotes the mean of distribution $G_n(X)$ then

$$m_n=\int_0^k\left[1-G_n\left(X\right)\right]dx=\frac{a_n}{\mu} \qquad \text{…(15)}$$

Clearly

$$a_n=\int_0^k\left[1-\left\{1-\exp\left(-x\right)^n\right\}\right]dx \qquad \text{…(16)}$$

From equation (6.16), we get

$$a_n=\begin{cases}0 & n=0\\ \sum_{i=1}^{n}1/i & n=1,2,\ldots\ldots,n\end{cases} \qquad \text{…(17)}$$

The mean waiting time of type B customer is given by

$$E\left(W_B\right)=\frac{1}{\mu}\left[\sum_{n=0}^{s}a_n+\sum_{n=s+1}^{N}\left\{\frac{n-s+1}{s}+a_{s-1}\right\}+\sum_{j=1}^{r-1}\sum_{jN+1}^{(j+1)N}\left\{\frac{n-\left(s+r\right)+1}{\left(s+j\right)}+a_{s+j-1}\right\}+\sum_{rN+1}^{K}\left\{\frac{n-\left(s+r\right)+1}{\left(s+r\right)}+a_{s+r}\right\}\right]P_n$$

...(18)

IV. PERFORMANCE INDICES

Equation (12) yields expected waiting time of type A customers as

…(19) Substituting the values of P_n from equation (4) in equation (18), we get expected waiting time of type B customers as

$$\mu E\left(W_B\right)=\Bigg[\sum_{n=0}^{s}a_n\frac{\rho^n}{n!}+\sum_{n=s+1}^{N}\left\{\frac{n-s+1}{s}+a_{s-1}\right\}\frac{\rho^n\mu^{n-s}\left(s^{\beta}\right)^{n-s}}{\left(s!\right)^{1-\beta}\left(n!\right)^{\beta}\prod_{i=s+1}^{n}\left\{s\mu+\left(i-s\right)\alpha\right\}}$$

$$+\sum_{j=1}^{r-1}\sum_{jN+1}^{(j+1)N}\left\{\frac{a-\left(s+j\right)+1}{\left(s+j\right)}+a_{s+j-1}\right\}$$

$$\frac{\rho^N\mu^{N-s}\left(s^{\beta}\right)^{N-s}\prod_{i=1}^{j-1}\left\{\left(s+i\right)^{\beta}\right\}^{n-jN}}{\left(s!\right)^{1-\beta}\left(n!\right)\prod_{i=s+1}^{N}\left\{s\mu+\left(i-s\right)\alpha\right\}\prod_{l=1}^{j-1}\prod_{i=lN+1}^{(l+1)N}\left\{\left(s+l\right)\mu+\left(i-\overline{s+l}\right)\alpha\right\}\prod_{i=jN+1}^{n}\left\{\left(s+j\right)\mu+\left(i-\overline{s+j}\right)\alpha\right\}}+\sum_{rN+1}^{K}\left\{\frac{a-\left(s+r\right)+1}{\left(s+r\right)}+a_{s+r}\right\}$$

$$\frac{\rho^N\mu^{N-s}\left(s^{\beta}\right)^{N-s}\prod_{i=1}^{r-1}\left\{\left(s+i\right)^{\beta}\lambda\right\}^{N}\left\{\left(s+r\right)^{\beta}\lambda\right\}^{n-rN}}{\left(s!\right)^{1-\beta}\left(n!\right)^{\beta}\prod_{i=s+1}^{N}\left\{s\mu+\left(i-s\right)\alpha\right\}\prod_{l=1}^{r-1}\prod_{i=lN+1}^{(l+1)N}\left\{\left(s+l\right)\mu+\left(i-\overline{s+l}\right)\alpha\right\}\prod_{i=rN+1}^{n}\left\{\left(s+r\right)\mu+\left(i-\overline{s+r}\right)\alpha\right\}}\Bigg]P_0$$

…(20)

The difference between mean waiting times for both types of customers denoted by D is given by

D = μ[E(w₁) – E(w₂)]

$$= \mu P_0 \left[\sum_{n=0}^{s} \left(a_{n+1} - a_n\right) \frac{\rho^n}{n!} + \sum_{n=s+1}^{N} \left(a_s - a_{s-1}\right) \frac{\rho^n \mu^{n-s} \left(s^\beta\right)^{n-s}}{\left(s!\right)^{1-\beta} \left(n!\right)^\beta \prod_{i=s+1}^{n} \left\{s + \mu + \left(i-s\right)\alpha\right\}} \right.$$

$$\left. + \sum_{j=1}^{r-1} \sum_{jN+1}^{(j+1)N} \left(a_{s+j} - a_{s+j-1}\right) \frac{\rho^N \mu^{N-s} \prod_{i=1}^{j-1} \left\{\left(s+i\right)^\beta \lambda\right\}^N \left\{\left(s+j\right)^\beta\right\}^{n-jN}}{\left(s!\right)^{1-\beta} \left(n!\right)^\beta \prod_{i=s+1}^{N} \left\{s\mu + \left(i-s\right)\alpha\right\} \prod_{l=1}^{j-1} \prod_{i=lN+1}^{(l+1)} \left\{\left(s+l\right)N + \left(i - \overline{s+l}\right)\alpha\right\} \prod_{i=jN+1}^{n} \left\{\left(s+j\right)\mu + \left(i - \overline{s+j}\right)\alpha\right\}} \right]$$

…(21)

V. PARTICULAR CASES

Case (i)

When β=1, r=0 and α=0 i.e., in case of no reneging and additional servers, the arrival rate reduces to

$$\lambda_n = \begin{cases} \lambda & ;0 \le n \le s \\ \left(\dfrac{s}{n+1}\right)\lambda & ;s \le n \end{cases} \qquad \text{…(22)}$$

By using equations (4) and (20), the mean waiting time for type A customers is given by

$$\mu E\left(w_2\right) = a_s + \sum_{n=0}^{s-1} \left(a_{n+1} - a_s - \frac{n-s+1}{s}\right) \frac{e^p}{n!} e^{-\rho} + \frac{e^{-\rho} + \left(\rho - s + 1\right)}{s} \qquad \text{….(23)}$$

On substituting the values of a_n from equations (15) and (23), the mean waiting time for type A customers is given by

$$\mu E\left(w_2\right)=\begin{cases}1+e+e^{-\rho}, & s=1\\ \dfrac{2+\rho+e^{-\rho}}{2}, & s=2\end{cases} \qquad \ldots(24)$$

We can also obtain difference D by using equations (21) as

$$D=\begin{cases}1 & for\, s=1\\ \dfrac{1+e^{-\rho}}{2} & for\, s=2\end{cases} \qquad \ldots(25)$$

Case (II)

When β=0, α=0 and r=0, then our model reduces to classical M/M/1 model with nopassing. In this case, we obtain

$$\mu E\left(w_2\right)=\begin{cases}\dfrac{1}{1-\rho} & for\, s=1\\ \dfrac{4+2\rho-\rho^2}{4-\rho^2} & for\, s=2\end{cases} \qquad \ldots(26)$$

And

$$D=\begin{cases}1 & for\, s=1\\ \dfrac{2}{2+\rho} & for\, s=2\end{cases} \qquad \ldots(27)$$

VI. NUMERICAL RESULTS

We have performed a numerical illustration to demonstrate of the effect of variation of different parameters on various performance measures. The numerical results are summarized in tables 1-4 and displayed in figures 1(i-iii) and 2(i-iii). In tables 1-4, we provide some performance measures namely expected waiting time $E(W_2)$ of type II jobs and difference (D) between expected waiting times of both type of jobs. In table 1, we note that D increases with r but $E(W_2)$ decreases by increasing the number of additional servers ® and traffic intensity (ρ). Table 2 gives the results when r, s, α and β are fixed and traffic intensity (ρ) and capacity size (N) vary. We observe that the expected waiting time $E(W_2)$ increases with ρ and N. In tables 3 and 4, the expected waiting time $E(W_2)$ and difference (D) decrease with the increase in α and β respectively.

The expected waiting time $E(W_2)$ as a function of traffic intensity (ρ) is plotted in figures 1(i) – 1(iii). By increasing ρ, the expected waiting time $E(W_2)$ increases for lower values of ρ while $E(W_2)$ varies considerably for larger values of ρ. The expected waiting time $E(W_2)$ also increases with the increase in the number of additional servers (r), threshold value (k) and reneging parameter (α). Figures 2(i) – 2(iii) demonstrate difference (D) between expected waiting time of both type of jobs vs. traffic intensity ρ by varying r, k, and α respectively. It is found that the difference (D) decreases by increasing ρ and asymptotically becomes zero as traffic intensity reaches 1.

VII. DISCUSSION

We have studied nopassing threshold queue with discouragement by incorporating removable servers. The provision of additional servers may be helpful in controlling the balking and reneging behavior of the customers. The cost associated with providing servers may be optimized by facilitating removable servers. The appropriate number of removable servers can be estimated by considering the expected waiting time of the customers.

ρ	***$E(W_2)$***			***D***		
	R=1	**r =2**	**r=3**	**r=1**	**r=2**	**r=3**
0.2	1.13	1.05	1.03	0.918	0.949	0.966
0.4	1.29	1.12	1.07	0.868	0.896	0.930
0.6	1.48	1.19	1.12	0.839	0.843	0.893
0.8	1.67	1.26	1.16	0.827	0.792	0.854
1.0	1.86	1.35	1.21	0.824	0.744	0.815
1.2	6.05	1.44	1.27	0.827	0.701	0.774
1.4	6.24	1.54	1.33	0.833	0.661	0.733
1.6	6.41	1.63	1.39	0.841	0.627	0.693
1.8	6.57	1.73	1.46	0.850	0.597	0.653

Table 1.: Performance measures with the variation of traffic intensity (ρ) and additional service positions (r) for s=2, α=0.6, β=0.4 and N=4.

ρ	***$E(W_2)$***			***D***		
	N=2	**N =4**	**N=6**	**N=2**	**N=4**	**N=6**
0.2	1.053	1.054	1.054	0.949	0.949	0.949
0.4	1.113	1.115	1.115	0.895	0.896	0.896
0.6	1.177	1.186	1.186	0.841	0.843	0.843

0.8	1.245	1.264	1.265	0.788	0.792	0.792
1.0	1.316	1.350	1.353	0.736	0.744	0.745
1.2	1.389	1.442	1.446	0.686	0.701	0.701
1.4	1.462	1.537	1.535	0.639	0.661	0.663
1.6	1.535	1.634	1.638	0.595	0.627	0.629
1.8	1.606	1.730	1.751	0.554	0.597	0.601

Table 2.: Performance measures with the variation of traffic intensity (ρ) and threshold value N for s=2, α=0.6, β=0.4 and r=4

A	***E(W₂)***			**D**		
	R=1	**R=2**	**r=3**	**r=1**	**r=2**	**r=3**
0.0	6.1432	1.2893	1.1631	0.8579	0.7868	0.8539
0.2	1.8895	1.2784	1.1627	0.8438	0.7891	0.8541
0.4	1.7552	1.2705	1.1623	0.8338	0.7908	0.8542
0.6	1.6697	1.2645	1.1620	0.8265	0.7920	0.8544
0.8	1.6095	1.2597	1.1617	0.8209	0.7931	0.8545
1.0	1.5645	1.2558	1.1614	0.8165	0.7939	0.8546

Table 3.: Performance measures with the variational of reneging parameter (α) and additional service position (r) for s=2, α=0.4, ρ =0.8 and N=4.

α	***E(W₂)***			**D**		
	R=1	**R=2**	**r=3**	**r=1**	**r=2**	**r=3**
0.0	1.7772	1.2708	1.1625	0.8332	0.7913	0.8543
0.4	1.6721	1.2653	1.1622	0.8269	0.7922	0.8544
0.8	1.6203	1.2618	1.1620	0.8235	0.7927	0.8545
1.2	1.5915	1.2594	1.1618	0.8215	0.7931	0.8546
1.6	1.5744	1.2577	1.1617	0.8203	0.7934	0.8547
6.0	1.5638	1.2565	1.1616	0.8195	0.7936	0.8548

Table 4.: Performance measures with the variation of balking parameter (β) and additional service positions (r) for s=2, α=0.6, ρ =0.8 and N=4.

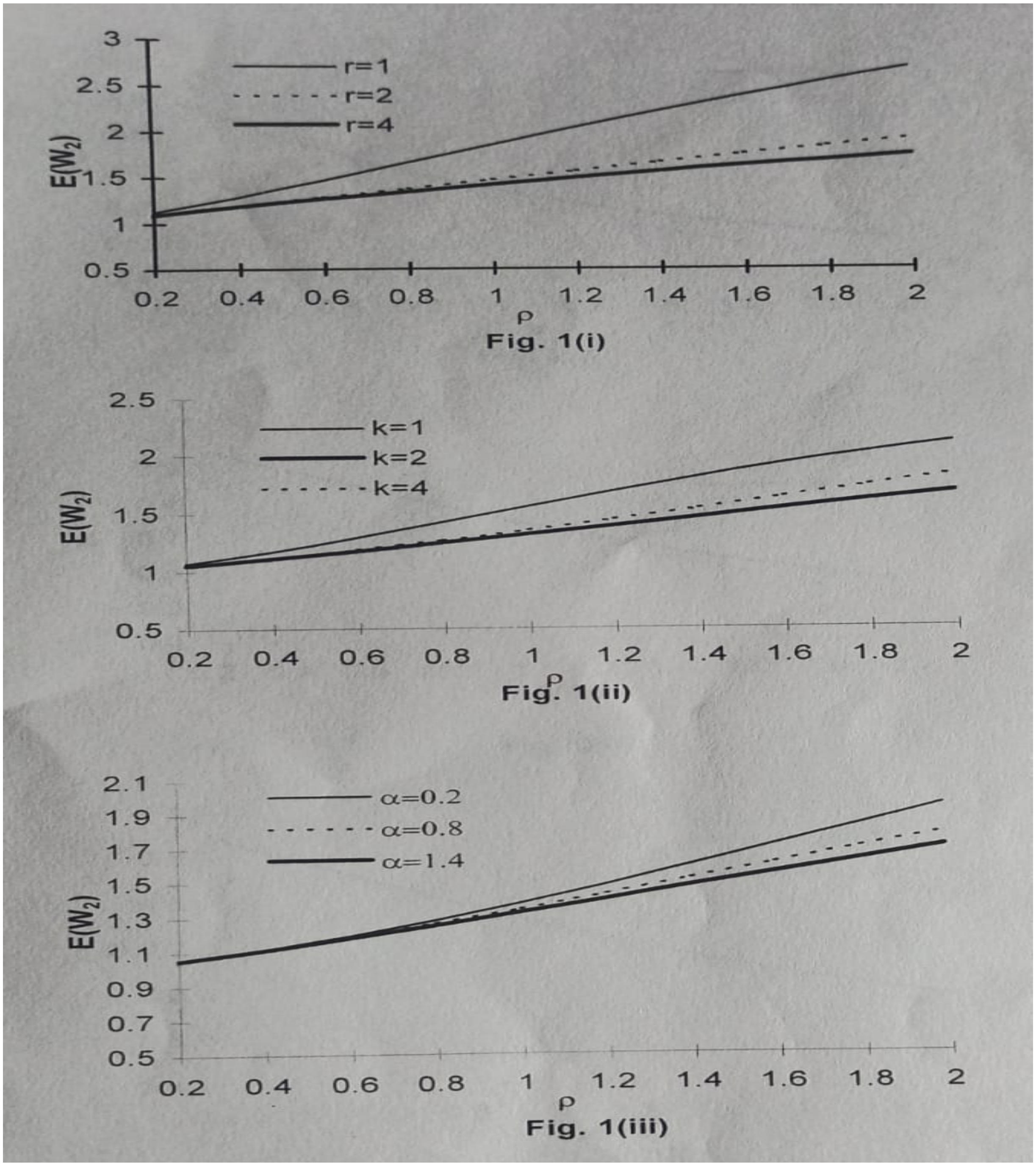

Fig. 1.: Expected waiting time for finite capacity model (i) by varying additional service positions (r) (ii) by varying threshold value (k) of jobs (iii) by varying reneging parameter (α).

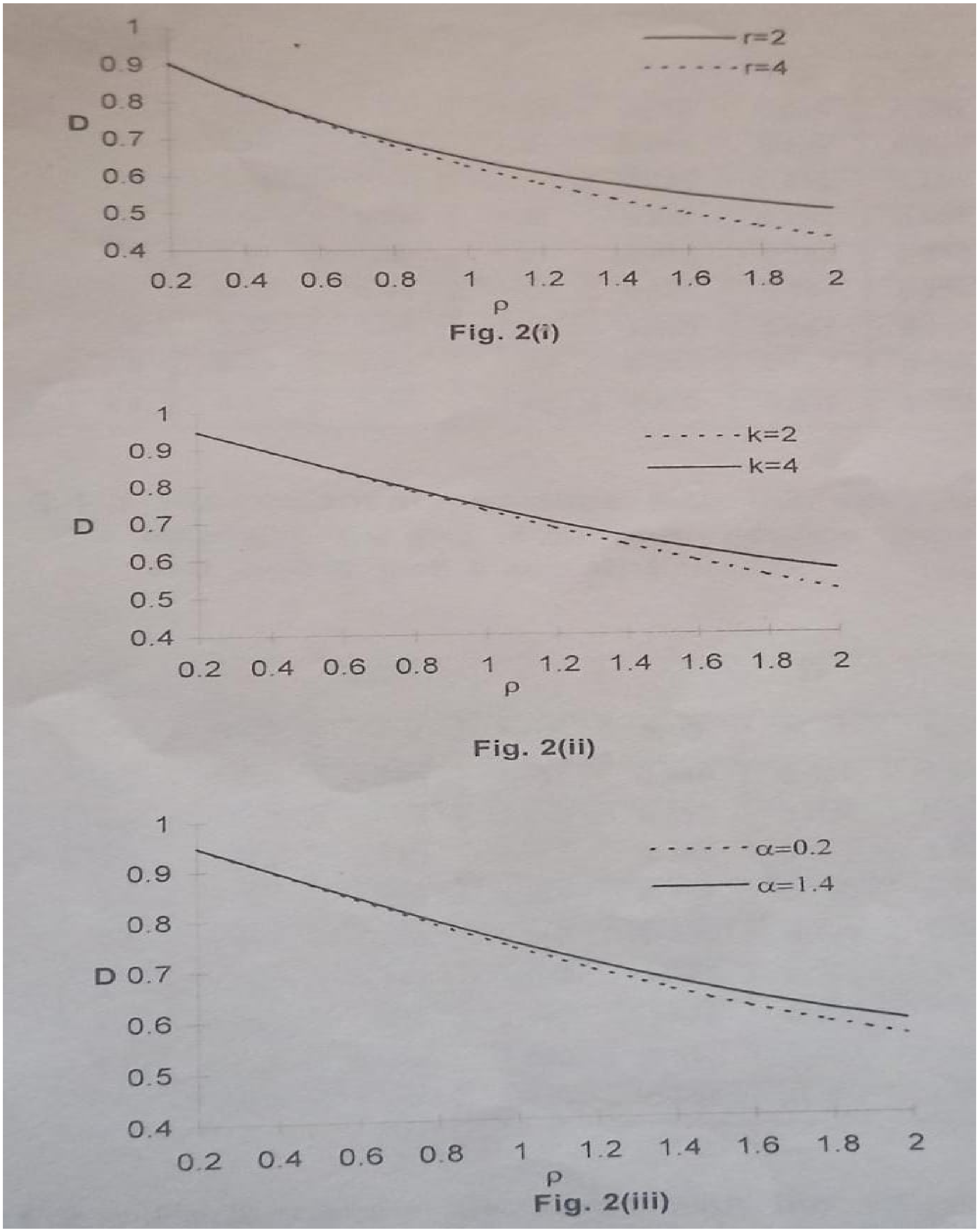

Fig. 2.: Difference for finite capacity model (i) by varying additional service positions (r) (ii) by varying threshold value (k) of jobs (iii) by varying reneging parameter (α).

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