Analysis of Neumann Boundary Condition on Newton Raphson Method

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Abstract: By the use of Neumann Boundary condition is solve the problem of Newton Raphson method it is demonstrated their application on nonlinear singular boundary element. In this research paper we are solve the Newton Raphson method at Neumann boundary condition [0,1] and its numerical problems are examined to improve the efficiency of this research work.

Keywords : Newton Raphson method , Neumann boundary condition, real valued functions , single variable functions.

***Mathematics Subject Classification*: 2020: 65Sxx, 54C30, 32F18.**

I. INTRODUCTION

The aim of this research paper is analyze and evaluate Newton Raphson method by the use of two point of nonlinear singular boundary valued function of Neumann boundary condition.

$$(p(x)y'(x))' = p(x)f\big(x,y(x)\big) \qquad 0 < x \leq 1, \tag{1}$$

subject to the Neumann boundary condition

$$y'(0) = 0 \ , \qquad \mu y(1) + \eta y(1) = \rho \tag{2}$$

where $\mu > 0$, $\eta \geq 0$ and ρ are finite constants

Newton Raphson Method

Newton Raphson method are derived from the name of Isaac Newton and Joseph Raphson it is root finding algorithm which provide the best approximation roots (zeros) of real valued function. It is single variable function x and defined for real variable x the functional derivative of f'.

Let $f(x) = 0$ be an algebraic or transcendental equation. Let x_0 be an approximate value of a root of the given equation .Let $x_0 + h = x_1$ be the exact root of $f(x) = 0$, where h is a very small quantity, then $f(x_0 + h) = 0$.

Expanding $f(x_0 + h)$ by Taylor's theorem

$$f(x_0 + h) = f(x_0) + hf'(x_0) + \frac{h^2}{2!}f''(x_0) + \cdots \ldots \ldots \ldots \ldots . = 0$$

Neglecting h^2 and higher powers of h(since h is a very small quantity)we obtain

$$f(x_0) + hf'(x_0) = 0$$

$$h = -\frac{f(x_0)}{f'(x_0)}.$$

∴ An approximate value of the root is given by

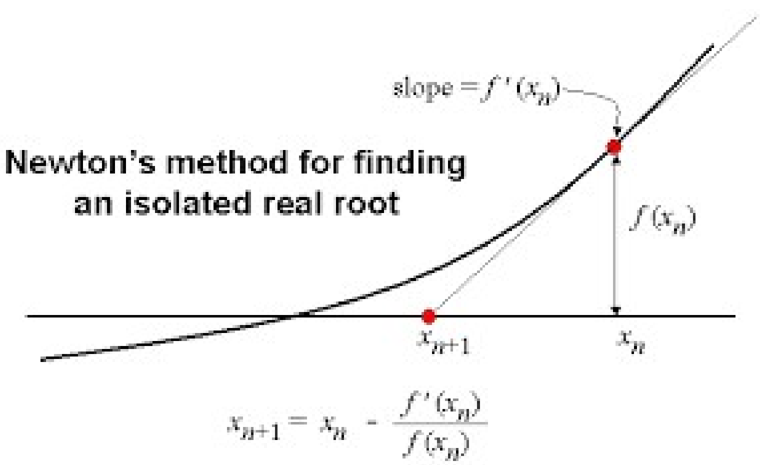
$$x_1 = x_0 + h \ , x_1 = x_0 - -\frac{f(x_0)}{f'(x_0)} \quad (1)$$

Replacing x_0 by x_1 and x_1 by x_2 in (1) , a better approximation x_2 of the root is given by

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}.$$

General formula of Newton Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



II. MAIN RESULT

By the use of Newton Raphson Method we also find out the Neumann boundary condition if root lies between [0,1] then it satisfy the Neumann boundary condition it illustrate by the example .

Example2.1. Use Newton Raphson Method to find a root of the equation $x^3 - 3x + 1 = 0$.

Solution. Let $f(x) = x^3 - 3x + 1 \therefore \quad f'(x) = 3x^2 - 3$

$f(0) = 1 = +ve$ and $f(1) = -1 = -ve$.

Therefore the root lies between 0 and 1.

Taking $x_0 = \frac{0+1}{2} = 0.5$

Now by Newton Raphson Formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^3 - 3n + 1}{3x_n^2 - 3} \qquad (1)$$

Putting $n = 0,1,2,3 \ldots \ldots$ successively in equation (1) ,we get

$$x_1 = x_0 - \frac{x_0^3 - 3x_0 + 1}{3x_0^2 - 3} = 0.5 - \frac{(-0.375)}{(-2.25)} = 0.3333.$$

$$x_2 = x_1 - \frac{x_1^3 - 3x_1 + 1}{3x^2 - 3}.$$

$$0.3333 - \frac{(0.0370459)}{(-2.666733)} = 0.3472 .$$

$$x_3 = x_2 - \frac{x_2^3 - 3x_2 + 1}{3x_2^2 - 3}.$$

Example 2.2: Find a real root of the equation $x = e^{-x}$ using the Newton Raphson method on Neumann boundary condition [0,1]

Solution. Now $f(0) = -1$ and $f(1) = 0.6321$ so that the root of the equation $x - e^{-x} = 0$, lies between 0 and 1.

From Newton- Raphson method , we have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n - e^{-x_n}}{1 + e^{-x_n}}$$

$$= \frac{(x_n + 1)e^{-x_n}}{1 + e^{-x_n}}$$

Let the initial approximation be $x_0 = \frac{0+1}{2} = 0.5.$

For $n = 0$, the First approximation x_1 is given by

$$x_1 = \frac{(x_0 + 1)e^{-x_0}}{1 + e^{-x_0}}$$

$$= \frac{(0.5 + 1)e^{-0.5}}{1 + e^{-0.5}}$$

$$= 0.56631$$

For $n = 1$, the second approximation x_2 is given by

$$x_2 = \frac{(x_1 + 1)e^{-x_1}}{1 + e^{-x_1}}$$

$$= \frac{(0.56631 + 1)e^{-0.56631}}{1 + e^{-0.56631}}$$

$$0.56714$$

For $n = 2$, the third approximation x_3 is given by

$$x_3 = \frac{(x_2+1)e^{-x_2}}{1+e^{-x_2}}$$

$$= \frac{(0.56714+1)e^{-0.56714}}{1+e^{-0.56714}}$$

$$= 0.56714.$$

Since $x_2 = x_3 = 0.56714$ therefore a real root of the given equation correct to five decimal places $= 0.56714$ and it is also satisfy the Neumann boundary condition.

II. RESULTS

In given example it is shown that the Newton Raphson Method satisfy the Neumann boundary condition [0,1] on the real root of the equation and we get better approximation up to three- two five places of decimal.

REFERENCES

[1] Y.Dong : One equivalent conditions for the solvability of equation (p(t)x′)′ +f(t,x) =h(t) satisfying linear boundary conditions with f restricted by linear growth conditions ,J.Math.Anal.Appl.245(2000)204–220.

[2] A.S. Householder: The Numerical Treatment of a Single Nonlinear Equation, McGraw-Hill,New York, 1970.

[3] Á H , YT Ng , Min C , Gibou F : Imposing mixed Dirichlet–Neumann–Robin boundary conditions in a level-set framework. Comput Fluids 2015;121:68–80 .

[4] Bramble JH , King JT : A finite element method for interface problems in domains with smooth boundaries and interfaces. Adv Comput Math 1996;6:109–38 .

[5] P. Sebah, X. Gourdon : Newton Õs method and high order iterations, numbers.computation.free.fr/Constants/constants.html, 2001.

[6] A.M. Wazwaz: A new algorithm for calculating Adomian polynomials for nonlinear operators,Appl. Math. Comput. 111 (2000) 53–69.