Color Class Dominating sets in Open Ladder and Slanting Ladder Graphs

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Abstract - Let $G = (V, E)$ be a graph. A color class dominating set of G is a proper coloring $\mathcal{C}$ of G with the extra property that every color class in $\mathcal{C}$ is dominated by a vertex in G. A color class dominating set is said to be a minimal color class dominating set if no proper subset of $\mathcal{C}$ is a color class dominating set of G. The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of G and is denoted by $\gamma_\chi(G)$. Here we obtain $\gamma_\chi(G)$ for Open ladder graph and slanting ladder graph.

Key words: Chromatic number, Domination number, Color class dominating set, Color class domination number.
Mathematics Subject Classification: 05C15, 05C69

I. INTRODUCTION

All graphs considered in this paper are finite, undirected graphs and we follow standard definitions of graph theory as found in [3].

Let $G = (V, E)$ be a graph of order p . The open neighborhood $N(v)$ of a vertex $v \in V(G)$ consists of the set of all vertices adjacent to v. The closed neighborhood of v is $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood $N(S)$ is defined to be $\cup_{v\in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$. For any set H of vertices of G, the induced sub graph $\langle H \rangle$ is the maximal subgraph of G with vertex set H.

A subset S of V is called a dominating set if every vertex in $V - S$ is adjacent to some vertex in S. A dominating set is a minimal dominating set if no proper subset of S is a dominating set of G. The domination number $\gamma(G)$ is the minimum cardinality taken over all minimal dominating sets of G. A $\gamma -$ set is any minimal dominating set with cardinality γ. A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The smallest number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by χ (G).

The join $G_1 + G_2$ of graphs G_1 and G_2 with disjoint vertex set V_1 and V_2 and edge sets E_1 and E_2 is the graph union $G_1 \cup G_2$ together with each vertex in V_1 is adjacent to every vertices in V_2. An Open ladder $O(L_p), p \geq 2$ is from two paths of length $n - 1$ with $V(G) = \{u_i u_{i+1}, v_i v_{i+1} \ /1 \leq i \leq n\} \cup \{u_i v_i \ /2 \leq i \leq n - 1\}$. A Slanting ladder graph SL_n is the graph obtained from two paths $u_1 u_2 \ldots \ldots \ldots u_n$ and $v_1 v_2 \ldots \ldots \ldots v_n$ by joining each u_i with v_{i+1}. In this paper, we obtain $\gamma_\chi(G)$ for Open ladder graph and slanting ladder graph.

II. MAIN RESULTS

Definition 2.1. Let G be a graph. A color class dominating set of G is a proper coloring $\mathcal{C}$ of G with the extra property that every color classes in $\mathcal{C}$ is dominated by a vertex in G. A color class dominating set is said to be a minimal color class dominating set if no proper subset of $\mathcal{C}$ is a color class dominating set of G. The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of G and is denoted by $\gamma_{\chi}(G)$.

Theorem 2.2. For the Open ladder graph O (L_p) $p \geq 2$,

$$\gamma_{\chi}(O(L_p))=\gamma_{\chi}(O(L_{2n}))=\begin{cases} \frac{2n}{3} & if\ n \equiv 0(mod3) \\ \left\lceil \frac{2(n-2)}{3} \right\rceil + 2 & otherwise \end{cases}$$

Let $L_p = L_{2n} = P_2 \times P_n$ where $n \geq 2$

Let $V\left(O(L_p)\right) = \{u_1, u_2, \ldots \ldots u_n, v_1, \ldots \ldots v_n\}$ and

$E\left(O(L_p)\right) = \{u_i, u_{i+1}, v_i v_{i+1} \quad / i = 1,2,3..(n-1)\} \cup \{u_i v_i \quad /2 \leq i \leq n-1\}$

We consider three cases

Case (i). When $n \equiv 0(mod3)$

Decompose L_p into $\frac{p}{3}$ copies of L_6. Assign $\frac{p}{3}$ distinct colors say $2i-1$ and $2i$ $(1 \leq i \leq \frac{p}{6})$ $\{u_{3i-2}, v_{3i-1}, u_{3i}\}$ and $\{v_{3i-2}, u_{3i-1}, v_{3i}\}$ respectively, we get a γ_{χ} – coloring. So $\gamma_{\chi}\left(O(L_p)\right) = \frac{p}{3} = \frac{2n}{3}$

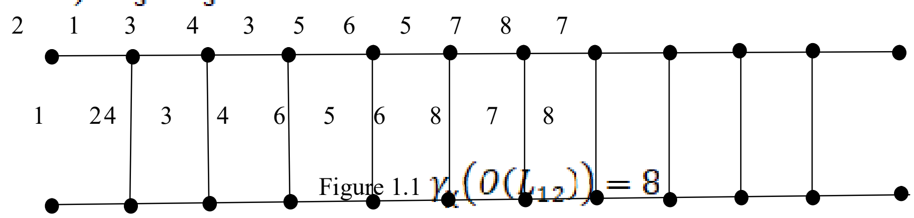

Figure 1.1 $\gamma_{\chi}\left(O(L_{12})\right) = 8$

Case (ii). When $n \equiv 1(mod3)$

Since $n - 1 \equiv 0(mod3)$, as in Case(i), $\gamma_{\chi}^{d}\left(L_{2(n-1)}\right) = \frac{2(n-1)}{3}$. Assign two distinct colors say $(\frac{2(n-1)}{3}) + 1$ and $(\frac{2(n-1)}{3}) + 2$ to the vertices $\{u_n\}$ and $\{v_n\}$ respectively, we get the required coloring. Thus $\gamma_{\chi}\left(O(L_p)\right) = \left\lceil \frac{2(n-2)}{3} \right\rceil + 2.$

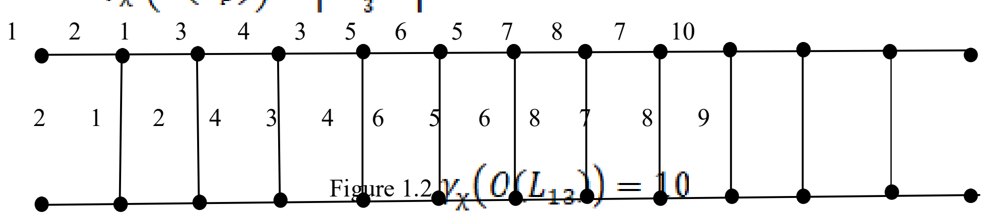

Figure 1.2 $\gamma_{\chi}\left(O(L_{13})\right) = 10$

Case(iii). When $n \equiv 2(mod3)$.

Since $n - 2 \equiv 0(mod3)$, as in case(i) $\gamma_{\chi}^{d}\left(L_{2(n-2)}\right) = \frac{2(n-2)}{3}$. Assign two distinct colors say

$(\frac{2(n-2)}{3}) + 1$ and $(\frac{2(n-2)}{3}) + 2$. So the vertices $\{u_{n-1}, v_n\}$ and $\{u_n, v_{n-1}\}$ respectively, we attain the γ_χ- coloring of L_p. Thus

$$\gamma_\chi(L_p) = \left\lceil \frac{2(n-2)}{3} \right\rceil + 2.$$

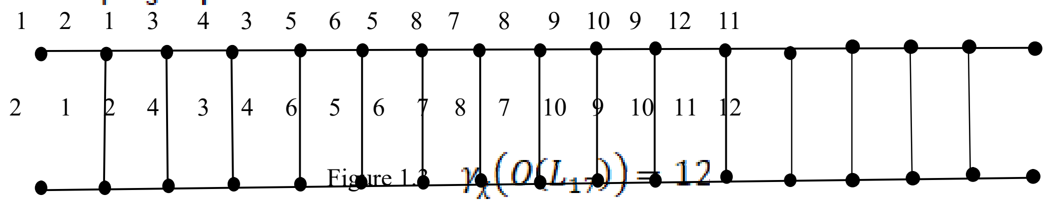

Figure 1.3 $\gamma_\chi(O(L_{17})) = 12$

Theorem 2.3. Let $SL_p = SL_{2n}$ be a Slanting ladder graph. Then

$$\gamma_\chi(SL_p) = \gamma_\chi(SL_{2n}) = \begin{cases} \left\lceil \frac{2(n-3)}{3} \right\rceil + 2 & if\ 2n \equiv 0(mod4) \\ \left\lceil \frac{2(n-1)}{3} \right\rceil + 1 & if\ 2n \equiv 2(mod4) \end{cases}$$

Proof. Let $SL_p = SL_{2n}\ (n \geq 3)$ be a Slanting ladder graph with $V(L_{2n}) = \{u_1, u_2, u_3 \dots \dots u_n, v_1, v_2 \dots \dots v_n\}$ and $E(G) = \{u_i u_{i+1}\ / i < n\} \cup \{v_i v_{i+1}\ /i < n\} \cup \{u_i v_{i+1}\ /1 \leq i \leq n-1\}$. We have two cases

Case(i). When $2n \equiv 0(mod4)$. We consider three subcases

Subcase 1.1 When $n \equiv 0(mod6)$For $i = 1,2, \dots \dots \lfloor \frac{n}{6} \rfloor$,

Let $H_i = < u_{6i-5}, u_{6i-4}, u_{6i-3}, u_{6i-2}, u_{6i-1}, u_{6i}$ $v_{6i-5}, v_{6i-4}, v_{6i-3}, v_{6i-2}, , v_{6i-1}, v_{6i} >$be the vertex induced subgraph of SL_{2n}. Then for each i, assign colors $4i-3,\ 4i-2,\ 4i-1$ and $4i$ to the vertices$\{u_{6i-5}, v_{6i-5}, v_{6i-3}\}$, $\{u_{6i-4}, v_{6i-4}, v_{6i-2}\}$, $\{u_{6i-3}, u_{6i-1}, v_{6i-1}\}$ and$\{u_{6i-2}, u_{6i}, v_{6i}\}$ respectively. We obtain a γ_χ– coloring of SL_{2n}. So $\gamma_\chi(SL_{2n}) = \left\lceil \frac{2(n-3)}{3} \right\rceil + 2$.

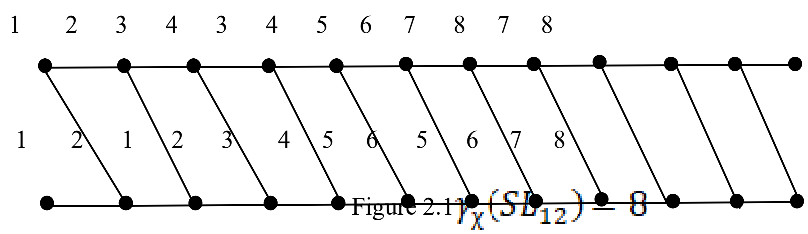

Figure 2.1 $\gamma_\chi(SL_{12}) = 8$

Subcase 1.2 When $n \equiv 2(mod6)$

Since $n-2 \equiv 0(mod6)$, as the same coloring of $SL_{2(n-2)}$ in Subcase 1.1, together with we assign two new colors say $\left\lceil \frac{2(n-3)}{3} \right\rceil + 1$ and $\left\lceil \frac{2(n-3)}{3} \right\rceil + 2$ to the vertices $\{u_{n-1}, v_{n-1}\}$ and $\{u_n, v_n\}$ respectively. We obtain γ_χ- coloring ofSL_{2n}. So $\gamma_\chi(SL_{2n}) = \left\lceil \frac{2(n-3)}{3} \right\rceil + 2$.

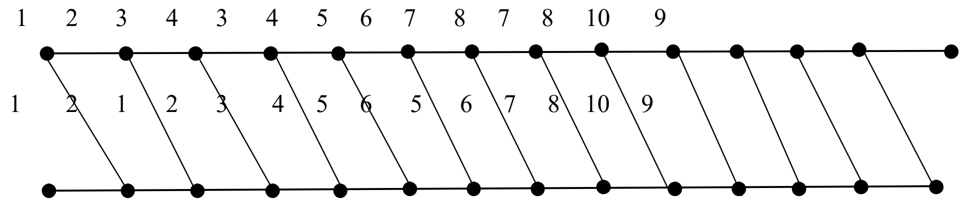

Figure 2.2 $\gamma_\chi(SL_{14})$=10

Subcase 1.3 When $n \equiv 4(mod6)$

Since $n-4 \equiv 0(mod6)$,by Subcase 1.1, $\gamma_\chi(SL_{2n})$ is obtained by $\gamma_\chi\left(SL_{2(n-4)}\right)$in addition with 4 new colors say, $2\left\lceil\frac{(n-3)}{3}\right\rceil+1$ and $2\left\lceil\frac{(n-3)}{3}\right\rceil+2$ to the vertices $\{u_{n-1}, v_{n-1}\}$ and $\{u_n, v_n\}$ respectively, to get a required γ_χ-coloring. Thus $\gamma_\chi(SL_{2n})$=$2\left\lceil\frac{(n-3)}{3}\right\rceil+2$

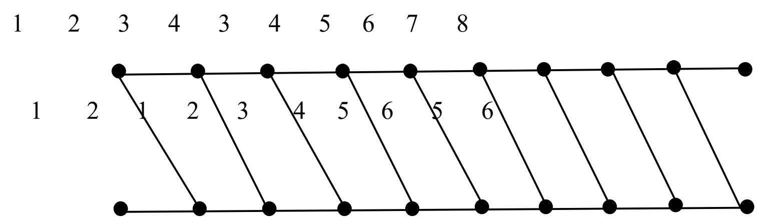

Figure 2.3 $\gamma_\chi(SL_{10})$=8

Case(2). When $2n \equiv 2(mod4)$

We consider 3 subcases

Subcase 2.1 When $n \equiv 1(mod6)$

Since $n-1 \equiv 0(mod6)$ By Subcase 1.1 $\gamma_\chi\left(SL_{2(n-1)}\right)$ together with a new color say $2\left\lceil\frac{(n-1)}{3}\right\rceil+1$ to the vertices $\{u_n, v_n\}$ to attain the γ_χ- coloring of SL_{2n}. So $\gamma_\chi(SL_{2n})$=$\left\lceil\frac{2(n-1)}{3}\right\rceil+1$.

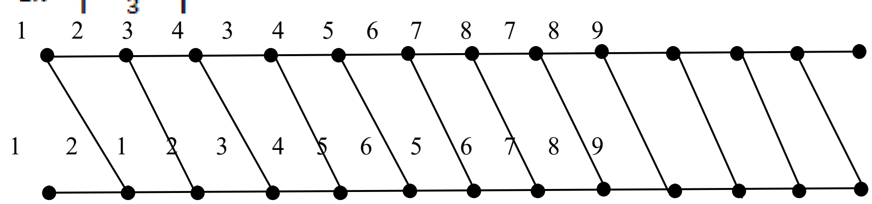

Figure 2.4 $\gamma_\chi(SL_{13})$=9

Subcase 2.2 When $n \equiv 3(mod6)$

By using Subcase 1.1 ,we obtain the coloring of $SL_{2(n-3)}$. Also we distribute three distinct colors say $\left\lceil\frac{2(n-1)}{3}\right\rceil-1$, $\left\lceil\frac{2(n-1)}{3}\right\rceil$and $\left\lceil\frac{2(n-1)}{3}\right\rceil+1$ to the vertices $\{u_{n-2}, u_n\}$, $\{u_{n-1}, v_{n-1}\}$ and $\{v_{n-2}, v_n\}$ respectively, to admit the γ_χ-coloring of SL_{2n}. So $\gamma_\chi(SL_{2n})$=$\left\lceil\frac{2(n-1)}{3}\right\rceil+1$

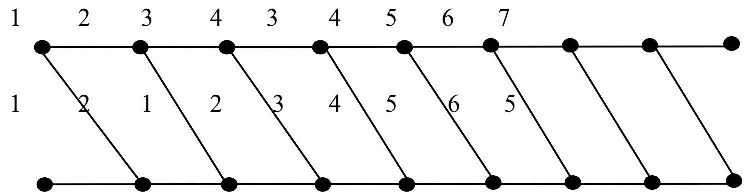

Figure 2.5 $\gamma_\chi(SL_9) = 7$

Subcase 2.3 When $n \equiv 5(mod6)$

Since $n-5 \equiv 0(mod6)$, SL_n is obtained by SL_{n-5} followed by SL_5 $\gamma_\chi(SL_n) = \gamma_\chi(SL_{n-5}) + \gamma_\chi(SL_5) = \left\lceil\frac{2(n-1)}{3}\right\rceil + 1$.

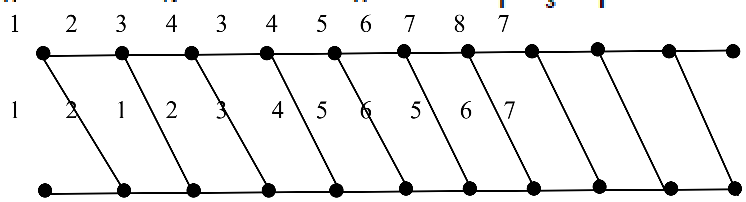

Figure 2.6 $\gamma_\chi(SL_{11}) = 8$

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