

Understanding Numerical Integration Techniques through examples from Electrical Engineering

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Abstract- With a view of providing better comprehension of the concepts underlying 1st order and 4th order numerical integration techniques, complete programs are written down using MATLAB m-code, which can be easily translated to C or python programming languages, if required. Initially an LC circuit is considered, followed by a Permanent Magnet Synchronous Generator (PMSG) based renewable energy generation system. It is shown that by simple techniques the computational time, can be reduced without sacrificing the accuracy. Avoiding use of ready-made software toolboxes and dealing with programming, makes the student grasp the concepts involved leading to critical thinking. Correlating the mathematical concepts with the specific topics of engineering specialization through related examples, makes reinforcement in learning thereby kindling the thought process, leading to creativity.

Keywords – Numerical integration methods: Euler’s technique, Runge-Kutta method, Resonance in Circuits, Electrical machine analysis, Renewable energy, Oscillating Water Column (OWC) based wave power generation.

I. INTRODUCTION

In the curriculum of the many branches of engineering, the subject of numerical integration of simultaneous ordinary differential equations forms either separate core/ elective course or finds place as few units of a mathematics paper. This is usually taught by teachers from Mathematics department, who may have difficulty in illustrating with examples relevant to the branch of study. Most of the B.E. specializations do have a course on basics of electrical engineering, detailing electrical circuits and machines. In addition, B.E. Electrical Engineering students have courses on performance and analysis of electrical machines and Special Electrical machines at a later stage of their studies. In between they would also study a course on signals and/or control systems theory. Initially while studying the mathematics of numerical integration, it is not possible for them to understand dynamics of electrical machines supplying independent loads(Muthukumar et al,2005) or connected to grid(Ravikiran et.al,2007), as they have not yet learnt the principles of their operation. In the simulation experiments, they are introduced to ready-made software toolboxes – therefore, there is only limited scope for in-depth understanding of underlying concepts.

To give the students better comprehension of solution of ordinary differential equations, (A single nth order differential equation can be split into n simultaneous first order equations, which can be put in canonical form), which forms defining equation for the electrical circuit / system, here, an attempt is made to apply simple numerical integration techniques. With the view of making the students correlate what they study in these courses, initially an LC resonating tank circuit is considered and then a Permanent Magnet Synchronous Generator (PMSG) based wind/wave energy extracting renewable energy system is considered. The program is written in MATLAB as a M-file instead of using toolboxes with built-in library components or demonstration examples available therein. This exercise makes the readers have better insight into the intricacies in the methods considered. The complete programs given, can be easily translated to other programming languages like C or python as needed.

The rest of the paper is organized as follows. Initially the classical example of an LC resonating tank circuit is considered in section II. Its theoretical analysis and simulated results based on First order Euler method and 4th order Runge-Kutta method are taken up. It is shown that by solving the defining equations of inductor and capacitor – one of them using forward difference algorithm and the other by backward difference algorithm, numerical stable results are obtained even with first order method, taking computationally lesser time. This idea becomes more important in solving for the dynamics of a PMSG driven by wind/wave turbine feeding a stand-alone load or feeding grid through a suitable grid- interactive power converting system. Considering the non-linear characteristics of the turbines in addition to the intermittent nature of the available green energy, deriving maximum energy from such renewable

energy source all the time, requires an overall optimization problem to be solved with solution of the turbine-machine set dynamics many times solved in the inner loop. Therefore, reduction of the computation time of the inner loop becomes vital. The PMSG fed stand-alone load case is considered first with a step change in mechanical input in section III. The actual case of PMSG fed from renewable energy source is considered in section IV, along with inferences derived and result discussions. Finally, the last section – section V, concludes the discussion. .

II. ANALYZING AN ELECTRICAL CIRCUIT

2.1 Theoretical analysis of an LC resonant tank circuit –

Considering an LC resonant tank circuit as shown in fig.1a, which is a conservative system, with no external input and with only initial conditions on either capacitor voltage and/or inductor current, the dynamics are obtained by solving using First order Euler method. In the circuit, the two elements L and C are both in series and in parallel. They are in series connection as the current through them are always equal. They are in parallel as the voltage across them are also always equal. This being a 2nd order system, as there are two energy storage elements, two state variables are to be identified. The inductor current and the capacitor voltage are the natural choice for the state variable assignment.

The defining equation for the capacitor is the Coulomb's law which states that $Q = CV$. Taking derivative on both sides, $dQ/dt = C dV/dt$. The current through the capacitor is dQ/dt . As the loop current assumed is in opposite direction so that it is same as the inductor current, $i = -dQ/dt = -C dv/dt$. The defining equation for the inductor is the Faraday's law which states that $v = d\psi/dt$, where, ψ is the flux linkage. The inductance L of coil is defined as the flux linkage per unit ampere of current and assuming that the reluctance of the flux path does not change with the value of current, this becomes, $v = L di/dt$. Here the voltage v is not only across the inductor but also it is the capacitor voltage due to circuit connections. Incidentally, the defining equation for the capacitor is the Kirchoff's Current Law (KCL) for the node formed by connecting the two components. Also, the defining equation for the inductor is the Kirchoff's Voltage Law (KVL) for the loop formed by connecting the two components.

It is found that the each of the state variable is proportional to the derivative of the other state variable. Therefore, a plot of one state variable against the other, also becomes phase portrait. Arranging the two equations in matrix form as, $dX/dt = A X$, where, $X = \{v, i\}$, leads to a 2x2 'A' matrix which has only off-diagonal elements. The roots of the characteristic equation are on the imaginary axis $\pm j \omega_r$, where the resonant frequency ω_r is $1/\sqrt{LC}$ rad/sec.

While teaching this in the circuit theory class, students may not have yet been introduced to control system basics, hence explaining in terms of poles of the transfer function, i.e, 'poles on the imaginary axis leading to marginal stability' may not be appealing to them. In some of the Universities, the syllabus of the subjects on the first year courses are such that these circuit theory concepts are to be learnt concurrently with Laplace transformation, or even before appreciating that 'the Laplace transformation can be used as a tool to convert differential equation into simple algebraic equation leading to easy solution of the time variations $i(t)$ and $v(t)$ '.

Solution of this classical LC resonant tank circuit is well-known, and hence it is taken-up here for initial comparison of the numerical methods. The students can be made to understand this case by considering 'auxiliary equation method', which is taught at school level itself. They know well that solution to the first order differential equation $dy/dt = y$, is given by $y = e^t$. The Taylor's series expression of the function e^x about the origin 0 by a deviation x, is $1 + x/1! + x^2/2! + x^3/3! + \dots + x^n/n! + \dots$. In this infinite series, the general nth term is $x^n/n!$, which when taken derivative with reference to x becomes the previous term. This type of easier explanation, rather than deriving based on limit of Δx tending to zero, make an impact on the learning behavior of the even average grasping type students. The solution to the second order differential equation $d^2y/dt^2 = y$, is also given by $y = e^t$, as the same concept can be applied repeatedly twice or even more times.

This can be further extended to the case of solving $dy/dt = kt$, for which the solution is $y = e^{kt}$. But if the given equation is second order differential equation $d^2y/dt^2 = ky$, (with k being a real and positive constant), the solution is $y = e^{+\sqrt{k}t}$. But, here the equation of the given circuit is of the form $d^2y/dt^2 = -ky$, (with k being a real and positive constant). This can be obtained by substituting the defining equation for the capacitor into the other equation of the inductor, in which case y here voltage 'v'. Or, this can also be obtained by substituting the defining equation for the inductor into the other equation of the capacitor, in which case y here current 'i'. This negative value '-k', makes $e^{+\sqrt{k}t}$ making the expression as $\sin(\sqrt{k}t)$ as per Euler's expansion for the 'e' raised to a 'complex' exponent. All engineering students would have learnt this solution through 'auxiliary equation method' for the case of equal roots, in the school level mathematics itself, though this explanation would be new to them.

Therefore, the solution to this LC resonating circuit is $v(t) = V_m \sin(\omega_r t + \theta_v)$ and $i(t) = I_m \sin(\omega_r t + \theta_i)$, where the values of amplitudes and phase angles do depend on the initial condition of this conservative system without any external inputs. For the case of having only initial condition on the capacitor voltage $v(0) = V_0$, and no initial inductor current as this circuit is closed only at $t=0$, the solutions are $v(t) = V_0 \cos(\omega_r t)$ and $i(t) = I_m \sin(\omega_r t)$, where the value

of peak current $I_m = V_0 \sqrt{C/L}$. As $\theta_v = \pi/2$, $\sin(\omega_r t + \theta_v)$ function is replaced with $\cos(\omega_r t)$. And $\theta_i = 0$. The expression for the peak current can be understood by applying ‘law of energy conversion’ to this lossless conservative system and equating the total energy at $t=0$, which is ‘ $\frac{1}{2} CV_0^2$ ’, to the total energy available only in the inductor after a quarter cycle of resonance, which is equal to ‘ $\frac{1}{2} LI_m^2$ ’. This LC tank circuit is ideal circuit used for explaining the concepts and in practical cases there would power dissipation due to winding resistance of the inductor and due to non-ideal nature of capacitor quantified by its ‘ $\tan \delta$ ’ factor. Therefore, sustained oscillations in the current and voltage is not possible and they die down after few cycles, even if considering the case of ‘under-damped response’, i.e, $R^2 < 4L/C$. The theoretical results for the ideal LC tank circuit is shown in fig.1.b. The values of the circuit components used and the initial conditions are also indicated therein.

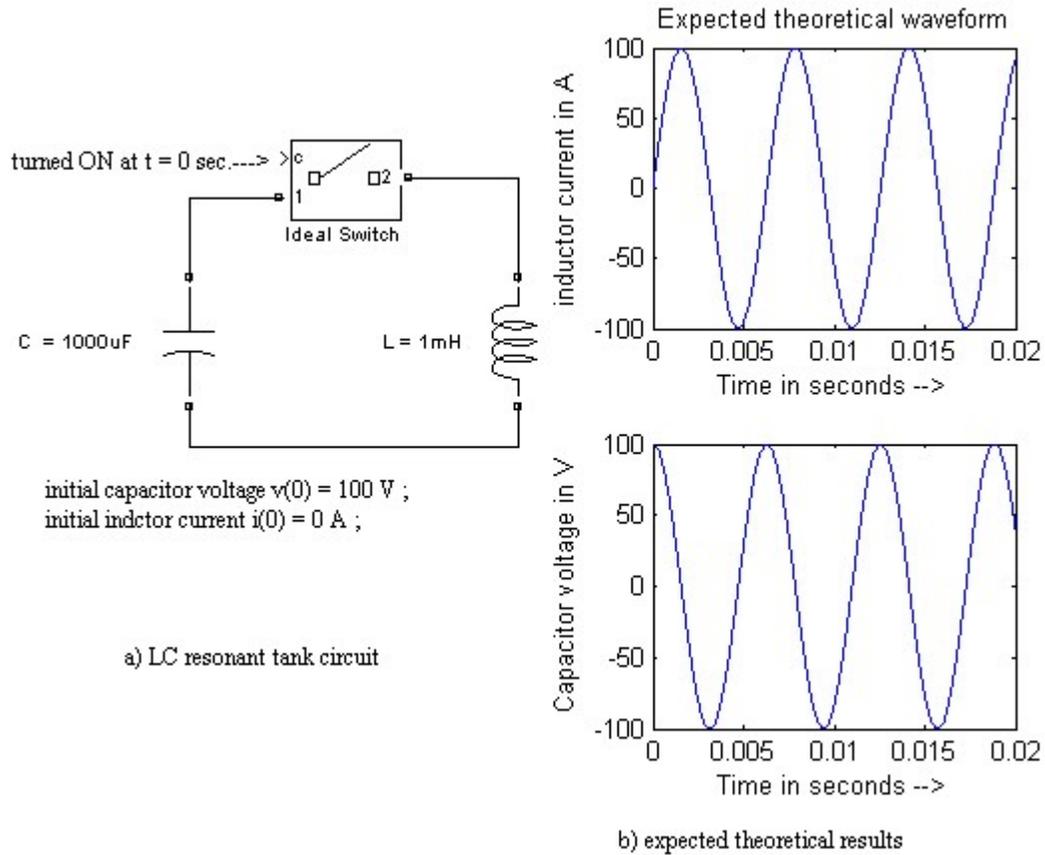


Figure 1. a) LC resonant tank circuit and b) theoretically expected results.

2.2 Solving by Numerical Integration by Euler 1st order method –

The complete MATLAB program for solving the simultaneous first order differential equations pertaining to this LC resonating tank circuit is listed below..

```

% for quick generation of space for workspace variables:
for ii = 1:1:2e4
    curr(ii) = 0.0; volt(ii) = 0.0 ;
end ;
% initial values
i_old=0; v_old=100; ind = 0.001 ; capa = 0.001 ;
h = sqrt(ind*capa) * 0.005; %time step for integration
% iteration by Euler's 1st order method
for ii=1:1:2e4
    inew = i_old + v_old/ind * h ;
    vnew = v_old - i_old/capa * h ;
    curr(ii) = inew;

```

```

volt(ii) = vnew;
i_old = inew;
v_old=vnew;
end;
subplot(2,2,1); plot(volt) ;
xlabel('Time in seconds-->');
ylabel('Capacitor voltage in V-->');
subplot(2,2,3); plot(curr) ;
xlabel('Time in seconds-->');
ylabel('inductor current in A-->');
subplot(2,2,2);
plot(volt,curr) ;
Title('results with Euler 1st order method');
ylabel('inductor current in A');
xlabel('Capacitor voltage in V');

```

The comment lines in the program make it self-explanatory. The results obtained are as given in fig.2 a) and b). The sinusoidal time-variations in both capacitor voltage and inductor current is noticed to be keeping on growing in amplitude. This is due to numerical instability of the forward difference algorithm followed by this Euler's 1st order method. This is against law of energy conservation as the total energy in the conservative system, i.e $\frac{1}{2} L \{i(t)\}^2 + \frac{1}{2} C \{v(t)\}^2$ is constant, therefore phase portrait has to be a circle as x –axis is v(t) and y-axis is i(t), both the peak values are equal in magnitude and so also the values of L and C. Therefore, the total energy = $\frac{1}{2} L \{i(t)\}^2 + \frac{1}{2} C \{v(t)\}^2 = \text{constant}$. This is in form $x^2 + y^2 = r^2$, which is the equation of a circle. The phase portrait based on theoretical results given in fig. 1.b) is given in fig.2.d).

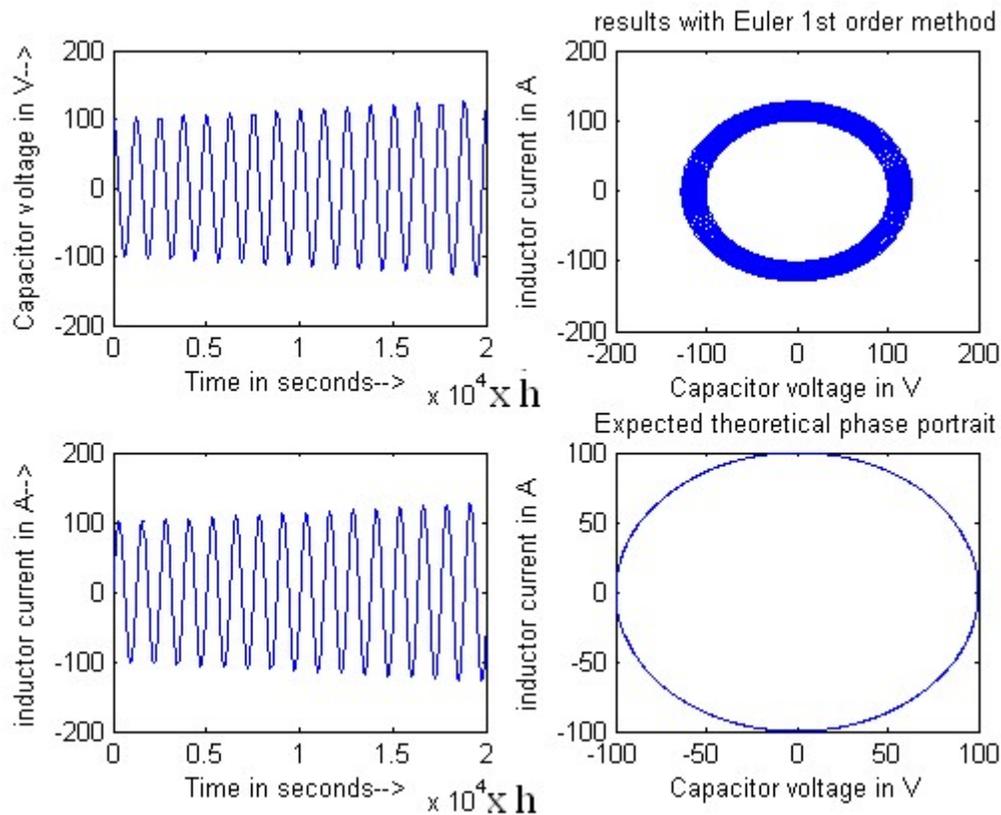


Figure 2. Results obtained by employing Euler's 1st order method a) & b) voltage and current waveform ($h=5 \times 10^{-6}$); c) phase portrait showing growing sinusoidal and instability of this forward difference method followed; d) Expected phase portrait based on waveform from fig.1.b

The ideal LC circuit considered has poles on the imaginary axis at $\pm j\omega_r$. But, the $s \rightarrow z$ mapping obtained by forward difference algorithms, i.e, s replaced by the $(z-1)/T_s$, map the imaginary axis $s = 0 + j\omega$ onto $z = 1 + j\Omega$, making all

marginally stable systems as unstable. If instead, backward difference algorithm is employed, the imaginary axis is mapped onto a circle of radius 0.5 with (0.5,0) as the center. This would make the marginally stable systems fully stable as all the points on the imaginary axis are mapped onto inside unit circle in the z-plane. The phase portrait obtained in that case is circle of shrinking radius, which is also wrong as it would be right for the case of RLC circuit with under-damped response. The simple first order methods lead to wrong results due to building -up of errors. This in turn, is owing to taking only upto first order term in the Taylor's series expansion. Instead, the popular fourth order Runge-Kutta method leads to better results, as it basically takes into account upto fourth order terms in the Taylor's series expansion around the point about which numerical integration is carried out. Next sub-section considers solving the same problem by this 4th order method.

2.3 Solving by Numerical Integration by Runge-Kutta 4th order method –

The MATLAB program for solving these simultaneous differential equations, by employing Runge-Kutta fourth order method is obtained by replacing lines 10 & 11 of the above program with the following program segment:

```

k1 = [v_old/ind, -i_old/capa ] * h ;
k2 = [(v_old+k1(2)/2)/ind, -(i_old+k1(1)/2)/capa ] * h ;
k3 = [(v_old+k2(2)/2)/ind, -(i_old+k2(1)/2)/capa ] * h ;
k4 = [(v_old+k3(2))/ind, -(i_old+k3(1))/capa ] * h ;
inew = i_old + (k1(1)+2*k2(1)+2*k3(1)+k4(1))/6;
vnew = v_old + (k1(2)+2*k2(2)+2*k3(2)+k4(2))/6;
```

With this modification the same program is executed and run to find correct expected results and phase portrait of circle just similar to figure 2.d. But as more evaluations are involved per iteration of the loop, it takes more computation time.

2.4 Computationally efficient algorithm for solving the same problem –

In the MATLAB code employing Euler method with forward difference algorithm, by simple replacement of *i_old* by *inew* in line 11, correct results are obtained without increasing the computation time at all. This can be understood from the fact that one of the differential equation is solved by equivalent 'forward difference equation' and the other being solved by equivalent 'backward difference equation' leading to cancellation of positive and negative errors. In any computer platform and any MATLAB version, the computational time taken by the Euler's method program with this change in the line 11, takes less than 'one third' of the time taken by the Runge-Kutta method based program.

This concept of using just first order method for solving of simultaneous equation is again tried out with a machine-load set in following section III. Computational time becomes more important here as this case has more involved dynamics owing to the acceleration/ deceleration of the inertia and mechanical time-constant concerned,

The LC circuit analyzed here forms the basis of modern resonant inverters/ converters, where the switching losses are eliminated by employing LC components, thereby avoiding the need for loss making snubber circuits. The program given above can be easily extended for analyzing a resonant based series inverter.

III. ANALYSIS OF PMSG FEEDING STAND ALONE LOAD

Electricity is not available as it is anywhere except for 'lightning', nor it is used anywhere as it is, except possibly in the mosquito squatter bat (killing the insect by electric shock). Electrical form of energy forms only the intermittent link, transfer bulk energy from energy centres (generated by power plants having alternators driven by turbines) to the load centres (industrial/ commercial or domestic consumers). For the renewable energy generation based on wind/ wave turbines, forming distributed generation system, previously induction generators were employed. Recently they are being replaced by the Permanent Magnet Synchronous Generators (PMSG) due to the disadvantage of Induction generators requiring reactive power support affecting voltage stability of the grid. Here, a PMSG feeding stand-alone load driven by an air-turbine is considered, with sudden step variation in the mechanical input power. Figure 3 a) shows the arrangement. The complete MATLAB program is listed below, alongside the defining equations of the PMSG and the characteristic equations for the air-turbine, are also provided for easy understanding of the program.

% pmsg transient simulation by phase variable model The machine variable based transient model

```

h = 0.0001 ; tfin = 0.2 ; tstep1 = 0.075 ; tstep2 = 0.15 ;
lss = 0.0012 ; lls = 0.0002 ; lm = ( lss - lls ) * 2/3 ;
rs = 0.4 ; motorconst = 0.075 ;
bm = 0.00002 ; inertia = 0.00007 ; pole = 4 ;
one = ( 2 * lss - lm ) / ( 2 * lss * lss - lss * lm - lm *
lm ) ;
two = lm / ( 2 * lss * lss - lss * lm - lm * lm ) ;
aa = rs * one ; bb = rs * two ; cc = - motorconst * one
; dd = - motorconst * two ; loadtorq = -1.5 ; rload = 10 ;
ias = 0 ; ibs = 0 ; ics = 0 ;
theta = 0 ; omega = 0 ; ii = 0 ;
for tt = 0 : h : tfin
    ii = ii + 1 ;
    vas = -ias * rload ; vbs = -ibs * rload ; vcs = -
ics * rload ;
    if ((tt >= tstep1) && (tt <= tstep2)) loadtorq = -1.0
; else loadtorq = -1.50 ; end ;
    diabydt = aa * ias + bb * ibs + bb * ics + cc *
omega * cos (theta) + dd * omega * cos(theta-2*pi/3) +
dd * omega * cos(theta- 4*pi/3) + one * vas +two * vbs
+ two * vcs ;
    dibbydt = bb * ias + aa * ibs + bb * ics + dd *
omega * cos (theta) + cc * omega * cos(theta-2*pi/3) +
dd * omega * cos(theta- 4*pi/3) + two * vas +one * vbs
+ two * vcs ;
    dwbydt = pole * pole * motorconst / 4 / inertia *
(ias * cos(theta) + ibs * cos(theta-2*pi/3) + ics *
cos(theta+2*pi/3)) - bm * omega / inertia - pole *
loadtorq / 2 / inertia ;
    dthetabydt = omega ;
    ias = ias + diabydt * h ; ibs = ibs + dibbydt * h ;
ics = -ias -ibs ;
    omega = omega + dwbydt * h ;
    theta = theta + dthetabydt * h ;
    if ( theta >= 2*pi) theta = theta - 2 * pi ; end ;
    aphcurr(ii) = ias ;
    bphcurr(ii) = ibs ;
    cphcurr(ii) = ics ;
    speed(ii) = omega ;
    position(ii) = theta ;
    time(ii) = tt ;
end ;
    
```

(Krause, P.C, et al., 2002) is employed for the PMSG:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} L_a & L_b & L_b \\ L_b & L_a & L_b \\ L_b & L_b & L_a \end{bmatrix} \begin{bmatrix} i'_a \\ i'_b \\ i'_c \end{bmatrix} + r_s \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \psi w \begin{bmatrix} \cos(\theta) \\ \cos(\theta - 2\pi/3) \\ \cos(\theta + 2\pi/3) \end{bmatrix}$$

where, $L_a = L_{ss}$ = stator self inductance per phase in H,

$L_b = -L_m / 2$, (L_m = stator mutual inductance per phase in H,)

ψ = machine constant in V-sec/rad,

w = speed in electrical rad/sec,

θ = angular position of the rotor in electrical radians,

v_a, v_b, v_c – are the stator phase voltages, and

i_a, i_b, i_c – are the stator phase currents; di/dt represented as i' .

In the program, the turbine torque is considered to be constant

With step changes at $t = 0.075$ second and 0.15 second.

Actually, the behavior of an air-turbine employed in an OWC based plant, is given by,

$$dP = Ca \times k_t \times \frac{1}{a} \times (V_x^2 + (r \times \omega_t)^2)$$

$$T_t = Ct \times k_t \times r \times (V_x^2 + (r \times \omega_t)^2)$$

Where, dP = Differential pressure in OWC chamber in N/m^2 ,

T_t = Gross torque produced by the impulse turbine in Nm.

And, V_x = absolute axial velocity of the air flow in m/sec,

ω_t = turbine speed in rad/sec.

The other parameters are: area of air-flow,

$a = 0.402 \text{ m}^2$, radius of blade tip, $r = 0.425 \text{ m}$, a

nd the turbine constant $k_t = 0.2103 \text{ kg/m}$.

Ca and Ct are input and torque co-efficients respectively,

both defined as a function of the flow co-efficient ϕ ,

which is the ratio $V_x / (r \times \omega_t)$.

The program employs the computationally efficient algorithm already established in previous section for the LC circuit. The results obtained with a step change in input power (which is the worst case) is considered. In the next section, the actual case of renewable energy fed air-turbine is considered, wherein intermittent natured input power with wide fluctuations is to be handled, still extracting the best possible energy. On the load side, supplying constant power to the stand-alone load without frequency variations requires embedding energy storage devices into the scheme. In case of connecting to the grid, grid-interactive inverters would be employed. Recently, Voltage Source Converters (VSC) are employed so as to have better power quality. The Indian Wave energy plant employed stand-alone load, typically a sea-water desalination plant supplying to the island. Hence the output of the PMSG is rectified into DC using a 3- Φ diode bridge. The water pumping does not demand making the power available to be consistently constant.

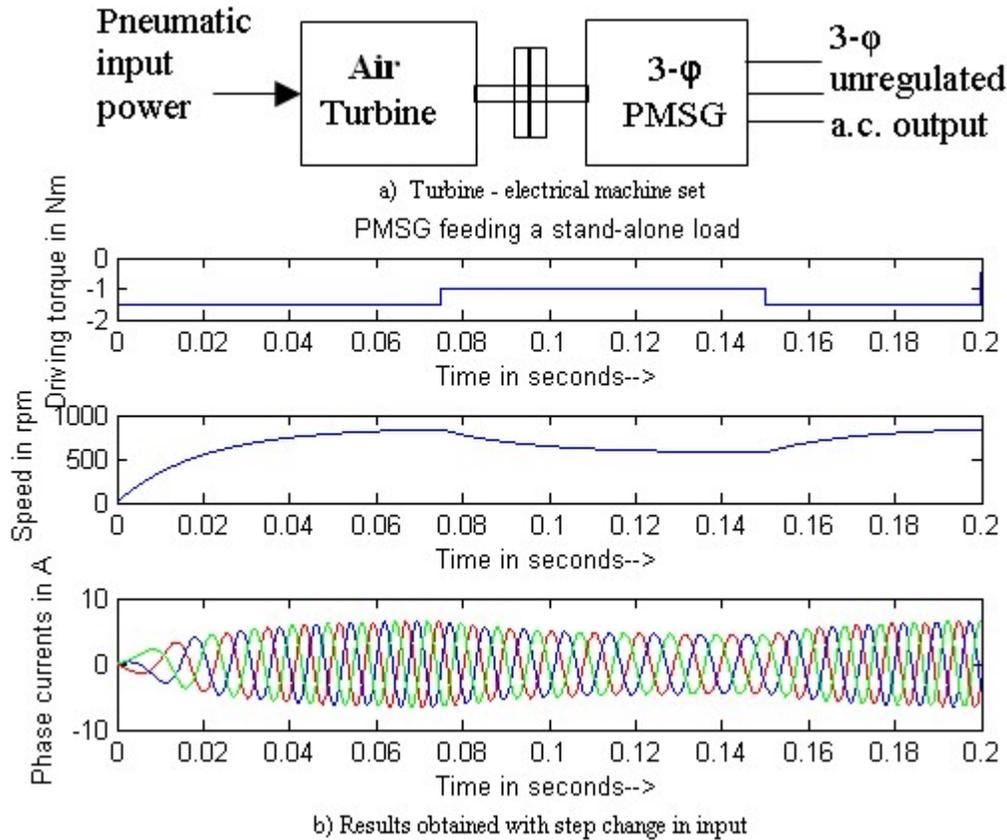


Figure 3. a) Turbine – machine set considered and b) Results obtained through numerical integration.

IV. FURTHER APPLICATION TO TURBINE ENERGY ENHANCEMENT SCHEME

The benefits of reduction in the computational time of the proposed technique without compromising the accuracy, is evident when this concept is applied to the case of Wave energy based renewable energy generation scheme. The Indian wave energy plant {based on Oscillating Water Column (OWC) concept} initially employed a Well's turbine as the air-turbine. Later it was replaced by the Linked Guide-vane type Impulse Turbine. Applying Maximum Power Point Trajectory (MPPT) concept here, is more complex and involved compared to the case of solar PV based power generation, owing to dependence of efficiency on the flow co-efficient. The flow co-efficient is defined as the ratio of axial velocity of the air-flow to the linear velocity of the blade-tip. Both of these air-turbines have fluctuations in the running speed with the ocean wave entering the Caisson and retarding back, due to self-rectifying action. Instead of maintaining a constant electrical load on the output of PMSG, by continuously adjusting the per-phase equivalent resistance reflected on the electrical side of the PMSG, it is possible to make the air-turbine operate for more part of time-interval near maximum efficiency conditions.

A maximum energy extraction scheme has been considered for the case of Linked Guide-vane based Impulse turbine case, and the results are readily available (Muthukumar S and Jayashankar V,2008). Though that Fixed Guide-vane based Impulse turbine exhibits lower efficiency ($\eta_{\max} = 39.13\%$ as tested with steady air-flow) when compared to Linked Guide-vane case ($\eta_{\max} = 55.36\%$ as tested with steady air-flow), the former type has advantage of giving

higher efficiencies over a wider range of flow co-efficient (for the range of 0.6 to 2.2, the efficiency > 35%), apart from requiring lesser on-site maintenance. Hence, only Fixed Guide-vane type impulse turbine was employed in the wave energy based desalination plant.

The maximum energy extraction scheme applied to the case of Fixed Guide-vane type impulse turbine, is not available in the literature and is carried out here, for the first-time and the results obtained are as given in fig. 4.

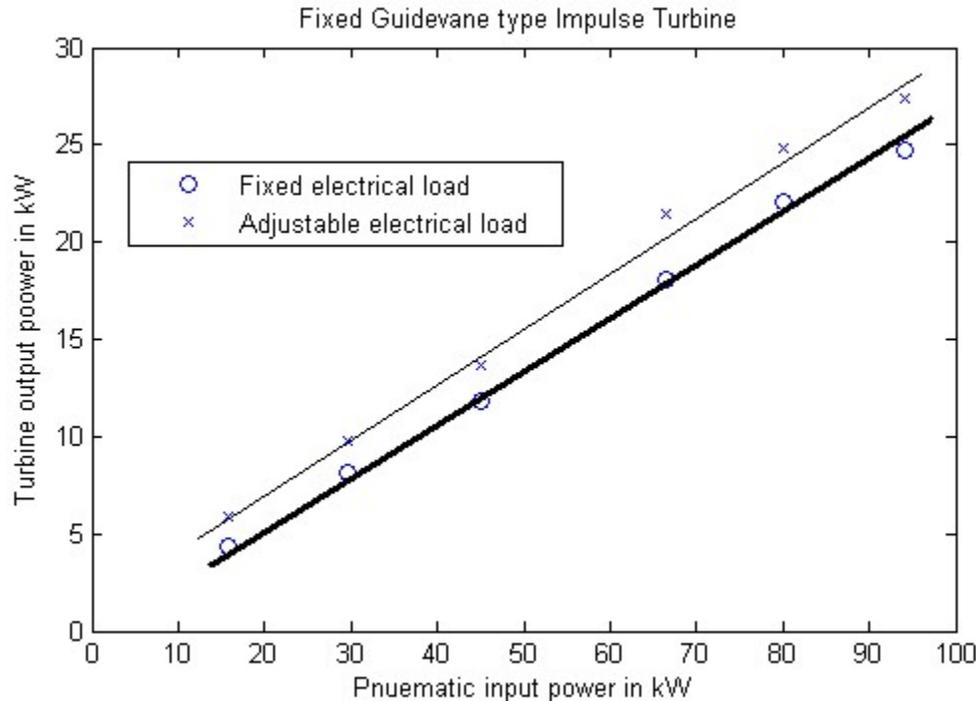


Figure 4. Results of 'Maximum energy extraction scheme' applied to FGV type Impulse turbine, showing increased turbine output power for the case of adjustable resistance load.

IV.CONCLUSION

Compared to using ready-made software toolboxes or using ODE solvers from built-in library of the application software, writing program employing C like programming languages, enhances the student perceptions of intricate mathematical techniques. MATLAB m-code based programming was taken up here due to its easy print/plot features. A computationally efficient technique for numerical integration of simultaneous differential equations is arrived at considering classical LC resonant circuit and its supremacy is established by using it for solving an air-turbine – machine set driven by renewable source and supplying to a stand alone load. A maximum energy extraction scheme has been applied to the case of Fixed Guide-vane type impulse turbine used in OWC based wave energy plant.

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