A Study of Peak Hours Optimization using Bulk Service Models

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Abstract - The paper deals with theinvestigation and analysis of the distribution of number of individuals waiting in queue using Erlangen techniques,the generating functions and the Laplace transforms. Here, the arrivals of units occur in groups according to a homogeneous Poisson process. The service time of customers is a random variable and is independent of the number of customers in the batch and try to solve real life problem.

I. INTRODUCTION

In queuing theory, a bulk queue isa general queuing modelwhere jobs arrive in are served in groups of random size. Network of such queues are known to have a product from stationary distribution under certain conditions. With heavy traffic conditions a bulk queue is known to behave like a reflected Brownian motion.

The busy period is the time interval which commences with an arrival of a person who activates the idle server and ends when for the first time thereafter, the server becomes idle again. The probability distribution of the number served in the busy period and duration of the busy period are of considerable interest.

There are various methods to study mathematically some aspects of the congestion theory. Neutus(1966) considered imbedded Markov and semi Markov analysis of a bulk queue. Cohen (1963) considered application of derived Markov chain in queuing theory. Jaiswal(1960) pointed out that from practical point of view it is preferable to find solution to a queuing problem by Erlangen procedure or its modification. Choudhary and Templeton (1972) analyzed the theory of bulk arrival and bulk service for some aspects of a single server queue.

A queue may be included under the title of a dual bulk queue if

(i) The service is in group following a probability distribution.

(ii) The arrivalsare in groups following a given probability distribution.

Hiller (1959) first initiated the analysis of a dual bulk queue that used the imbedded Markov chain technique to discuss some properties related to the dual bulk queue. Kailson (1962) used the phase space method. Bhat (1964) used the results from fluctuation theory to analysis the equilibrium distribution of the queue length and busy period.

Rao (1968) employed the phase method or a modified Erlangen procedure to consider queue length for a transportation type dual bulk queue with an arbitrary arrival time distribution and exponential service time distribution.

Here we have presented some aspects of a single server dual bulk queue by the modified Erlangen procedure in a general manner.

General Erlangen Model is a general queuing model in which the arrival and service rates depend upon the length of the queue. Some persons desiring services may not join the queue since it is too long, thus affecting the arrival rate. And service rate is also affected by the length of the queue.

II. ASSUMPTIONS AND NOTATIONS

2.1 Kendall's notation:

In Kendall's notation for single queuing nodes, the random variable denoting bulk arrivals or services is denoted with a superscript, for example,

$M^x|M^y|1$ denotes an M|M|1 queue where the arrivals are in batches determined by the random variable X and service in bulk is determined by the random variable Y.

2.2 Bulk Service:

Customers arrive at random instants according to a Poisson process and form a single queue, from the front of which batches of customers are served at a rate with independent distribution. The equilibrium distribution, mean and variance of queue length are known for this mode. Markov decision process can be used to optimize maximum size of batch, subject to operating cost constraints.

2.3 Bulk Arrival:

Optimal service provision procedure to minimize long run expected cost.

2.4 Arrival Process:

The arrival of units occurs in groups according to a homogenous Poisson process with arrival rate **λ.** The group size X is random variable and,

a_r=P{ X^m . r} , r=1,2,3

It is also assumed that the distribution for the number of customers who arrive at each arrival period of notable eventhave finite mean ā and C_a^2 with generating function.

$$A(X)=a_m X^m \qquad \sum_{l}^{\infty}$$

2.5 Queue Discipline:

The queue discipline is first come first out. This means customers first entering into the queue will get out of the queue into the service channel for service. The order in which the members of a particular group present themselves to the server is a matter of insignificance.

2.6 Service Mechanism :

The service time of customers is a random variable having a probability distribution function

$$\sum^{i} C_r \frac{\mu (\mu t)^{r-1}}{(r-1)!} e^{-\mu t}$$

andis independent of the customers in the batch. C_r, r=1,2,3….. being the probability that a batch of S or the entire queue which is lesser taken in the rth phase each having exponential service time with a parameter μ. Itis further assumed that among the customers who arrive where server is idle, S are taken immediately for service if their number of arrivals is greater than S, otherwise all are taken for service.

2.7 Traffic Intensity:

Let the arrival and service assumptions be expressed in terms of the traffic intensity

$$\frac{\lambda \bar{a} \Sigma r C_r}{\mu_s}$$

which will be assumed to be less than unity in the case of steady state.

III. DERIVATION OF THE EQUATIONS

Here we have,

$P_{n,r}(t)$ = Probability that there are n customers waiting in the queue and the service is in the rth phase at time t.

$P_o(t)$ = Probability that there is no customers waiting in the queue and the server is free i.e. the system is empty at time t.

The difference differential equations for the queuing procedure are as follows:

$P'_o(t) = -\lambda\, P_o(t) + \mu\, P_{o,1}(t)$

$P'_{o,r}(t) = -(\lambda+\mu)P_{o,r}(t) + \mu P_{o,r+1}(t) + \mu C_r \sum_{i=1}^{S} P_{i,1}(t) + \lambda C_r \sum_{i=1}^{S} a_i P_0(t)$

$P'_{o,j}(t) = -(\lambda+\mu)P_{o,j}(t) + \mu C_j \sum_{i=1}^{S} P_{i,1}(t) + \lambda C_j \sum_{i=1}^{S} a_i P_0(t)$

$P'_{n,r}(t) = -(\lambda+\mu)P_{n,r}(t) + \lambda \sum_{m=1}^{n} a_m P_{n-m,r}(t) + \lambda P_{m,r+1}(t) + \mu C_r P_{n+s,1}(t) + \lambda C_r$

$a_{n+s} P_0(t)$ $(n>0, \leq r<j)$

$P'_{n,j}(t) = -(\lambda+\mu)P_{n,j}(t) + \lambda C_j \sum_{m-1}^{n} a_m P_{n-m,j}(t) + \mu C_j P_{n+s,1}(t) + + \lambda C_j a_{n+s} P_0(t)$

$,n>0$, where $P'_{n,r}(t) = \frac{d\{P_{n+s}(t)\}}{dt}$ etc.

1. Solution of Equations:

We denote the Laplace transform of $P_{n,r}(t)$ by $P'_{n,r}(\alpha)$. Then

$P'_{n,r}(\alpha) = \int_0^{\infty} exp.(-\alpha t) Pn,r\,(t)\; dt$

Also, let us assume that the process starts with the system being empty, i.e.

$P_0(0)=1$

We further introduce the following generating function

$A(\gamma) = \sum_{r=1}^{\infty} a_r \gamma^r$

$P_n(\gamma,\alpha) = \sum_{r=1}^{j} \bar{P}_{n,r}(\alpha)\gamma^r$

$H(\gamma,X,\alpha) = \sum_{n=0}^{\infty} P_n(\gamma,\alpha)X^n = \sum_{n=0}^{\infty} \sum_{r=0}^{j} \bar{P}_{n,r}(\alpha)X^n\gamma^r$

Taking the Laplace transforms of the basic equation multiplying by the appropriate powers of X and γ and summing out the values of n and the following equation is obtained:

$$\left[\alpha+\lambda+\mu-A(X)-\frac{\mu}{y}\right]H(\gamma,X,\alpha)+\mu\left[1-\frac{C(y)}{X^S}\right]\sum_{n=0}^{\infty}\bar{P}_{n,1}(\alpha)X^n - -$$

$$\left[\frac{C(y)}{X^S}\right]\left[\sum_{i=1}^{s-1}\left\{\mu\bar{P}_{i,1}(\alpha)+\lambda a_1\bar{P}_0(\alpha)\right\}\right](X^s-X')+1-(\alpha+\lambda-\lambda A(x)\bar{P}_{0(\alpha)})=0$$

……(1)

Where $\sum_{r=1}^{j} \quad \sum_{n=1}^{\infty} \quad \sum_{m=1}^{n} a_m P_{n-m,r}(\alpha) X^{n-m} y^r = A(X)\, H(\gamma, X, \alpha)$

And $C(\gamma) = \sum_{r=1}^{j} C_r \gamma^r$

The above equation valid for |X|<1 and |Y|<1, it is also valid for

$\gamma = \mu/\ \alpha+\lambda+\mu-\lambda\Lambda(X)$, because $R_e(\alpha)>0$.

Thus we get from $\sum_{n=0}^{\infty} P_{n,1}(\alpha) X^n =$

$$\frac{\sum_{i=1}^{S-1}\{\mu\bar{P}_{i,1}(\alpha)+\lambda a_1 \bar{P}_0(\alpha)\}(Xs - X') + 1 - (\alpha+\lambda-\lambda A(x)\bar{P}_0(\alpha))}{\mu\left[\frac{X^S}{C(y)} - 1\right]}$$

……..(2)

where γ is given by $\gamma = \mu/\ \alpha+\lambda+\mu-\lambda\Lambda(X)$

if we denote $C(\mu/\ \alpha+\lambda+\mu-\lambda\Lambda(X))$ by B(x)

and $\sum_{n=0}^{\infty} \bar{P}_{n,1}(\alpha) X^n$ by $e_j(X,\alpha)$, then by putting (2) in (1) we get after substituting $\gamma=1$, $H(1,X,\alpha) = \sum_{n=0}^{\infty} P_{n,1}(\alpha) X^n$

…….(3)

$H(1,X,\alpha) = \frac{\mu\left(\frac{1}{E(X)}-1\right)}{\alpha+\lambda-\lambda_j\Lambda(X)} G(X,\alpha)$ where $P_n(1,\alpha) \sum_{r=1}^{j} \bar{P}_{n,r}(\alpha)$

is the Laplace transform of the probability that there are n units waiting in the queue at time t.H(1,X,α) is the generalization of the result obtained by Jaiswal(1960).

V. CONCLUSION

The result may be applied to batch arrivals. A general class of queue having simultaneous arrival with arbitrary independent service distributions has also been considered. There are many queuing activities in which arrivals and service can be in a group i.e. in batches or in bulk. Several people may go to a restaurant together and obtain service as a batch. A number of long-distance telephone calls may present themselves simultaneously before an operator. The result can be applied in these types of queuing problems. Scheduling of mechanical transport fleets, works in production control, minimization of congestion due to traffic delay at tool booths.

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