Some Results On Lacunary Ideal limit Point and Cluster Points of the Sequences of Fuzzy Real Numbers

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Abstract- In this article, the concepts of lacunary Ideal limit point and cluster point-of a multiple sequences of fuzzy real numbers having multiplicity greater than two is introduced and some equivalence results are established.
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I. INTRODUCTION

After the introduction of fuzzy set by L. A. Zadeh [36] in 1965 different types of fuzzy real-valued sequence spaces have been introduced and studied by several mathematicians by using the notion of fuzzy real numbers. Agnew [1] studied the summability theory of multiple sequences and obtained certain theorems which have already been proved for double sequences by the author himself. In order to generalize the idea of convergence of real sequences, Kostyrko, Šalát and Wilczyński [17] introduced the idea of ideal convergence for single sequences in 2000-2001. Later on it was further developed by Šalát et al. ([18], [27]), Kumar and Kumar [20], Tripathy and Tripathy [35], Das et al. [6], Sen and Roy [30], Nath and Roy [21], Nath and Roy [22] and many others.

The different types of notions of multiple sequences was introduced and investigated at the initial stage by Sahiner et al. [26], Kumar et al. [19], Dutta et al. [8], Savas and Esi [29], Esi ([11], [12]). Some more works on fuzzy triple sequences are found in ([23], [24]).

A lacunary sequence is an increasing integer sequence $\theta = \langle k_r \rangle \quad (r = 0, 1, 2, 3, ...)$ of positive integers such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be defined by $J_r = (k_{r-1}, k_r]$ and the ratio $\frac{k_r}{k_{r-1}}$ will be defined by q_r.

Friday and Orhan [13] introduced the concept of lacunary statistical convergence in 1993. Different classes of lacunary sequences have been studied by some renowned researchers. Nuray [25], Demirci [7], Bligin [5], Altin et al. ([2], [3]), Altin [4], Gokhan et al. [14], Subramanian and Esi [31], Esi [10], Savas [28], Tripathy and Baruah [32], Dutta et al. [9], etc. Are some of them.. The concept of lacunary I-convergence was introduced in [33]. More works on lacunary I-convergence was found on ([15], [16],[34]) etc.

A fuzzy real number on R is a mapping $X : R \to L(=[0,1])$ associating each real number $t \in R$ with its grade of membership X (t).Every real number r can be expressed as a fuzzy real number $\bar{r}$ as follows:

$$\bar{r}(t) = \begin{cases} 1 & if \quad t = r \\ 0 & otherwise \end{cases}$$

The α-level set of a fuzzy real number X, $0 < \alpha \le 1$, denoted by $[X]^\alpha$ is defined as
$[X]^\alpha = \{t \in R : X(t) \ge \alpha\}$.

A fuzzy real number X is called convex if $X(t) \ge X(s) \wedge X(r) = \min(X(s), X(r))$, where $s < t < r$. If there exists $t_0 \in R$ such that $X(t_0) = 1$, then the fuzzy real number X is called normal. A fuzzy real number X is said to be upper semi-continuous if for each $\varepsilon > 0$, $X^{-1}[0, a + \varepsilon))$, for all $a \in L$ is open in the usual topology of R. The set of all upper semi continuous, normal, convex fuzzy number is denoted by $R(L)$. The additive identity and multiplicative identity in $R(L)$ are denoted by $\overline{0}$ and $\overline{1}$ respectively.

Let D be the set of all closed bounded intervals $X = [X^L, X^R]$ on the real line R. Then $X \leq Y$ if and only if $X^L \leq Y^L$ and $X^R \leq Y^R$. Also let $d(X,Y) = \max(|X^L - X^R|, |Y^L - Y^R|)$.

Then (D,d) is a complete metric space.

Let $\bar{d} : R(L) \times R(L) \to R$ be defined by $\bar{d}(X,Y) = \sup_{0 \leq \alpha \leq 1} d([X]^\alpha, [Y]^\alpha)$, for $X, Y \in R(L)$.

Then $\bar{d}$ defines a metric on $R(L)$.

Let X be a non empty set. A non-void class $I \subseteq 2^X$ (power set of X) is said to be an ideal if I is additive and hereditary, i.e. If I satisfies the following conditions:

(i) $A, B \in I \Rightarrow A \cup B \in I$ and (ii) $A \in I$ *and* $B \subseteq A \Rightarrow B \in I$.

A non-empty family of sets $F \subseteq 2^X$ is said to be a filter on X if

(i) $\emptyset \notin$ F (ii) A, B $\in$ F $\Rightarrow$ A $\cap$ B $\in$ F and (iii) A $\in$ F and A $\subseteq$ B $\Rightarrow$ B$\in$ F.

For any ideal I, there is a filter F(I) given by $F(I) = \{ K \subseteq N : N \setminus K \in I \}$.

An ideal $I \subseteq 2^X$ is said to be non-trivial if I$\neq\emptyset$ and X $\notin$ I. Clearly $I \subseteq 2^X$ is a non-trivial ideal if and only if $F = F(I) = \{X - A : A \in I\}$ is a filter on X.

A non-trivial ideal I is called admissible if and only if $\{\{n\} : n \in N\} \subset I$. A non-trivial ideal I is maximal if there cannot exists any nontrivial ideal $J \neq I$ containing I as a subset.

A subset E of $N \times N \times N$ is said to have density $\delta(E)$ if $\delta(E) = \lim_{p,q,r \to \infty} \sum_{n=1}^{p} \sum_{l=1}^{q} \sum_{k=1}^{r} \chi_E(n,l,k)$ exists where χ_E is the characteristic function of E.

Throughout the article, the ideals of $2^{N \times N \times N}$ will be denoted by I_3.

Example 1.1. Let $I_3(\rho) \subset 2^{N \times N \times N}$ i.e. The class of all subsets of $N \times N \times N$ of zero natural density. Then $I_3(\rho)$ is an ideal of $2^{N \times N \times N}$.

Example 1.2. Let $I_3(P)$ be the class of all subsets of $N \times N \times N$ such that $D \in I_3(P)$ implies That there exists $m_0, n_0, l_0 \in N$ such that

$D \subseteq N \times N \times N - \{(m,n,l) \in N \times N \times N : m \geq m_0, n \geq n_0, l \geq l_0\}$.

Then $I_3(P)$ is an ideal of $2^{N \times N \times N}$.

II. PRELIMINARIES AND BACKGROUND

In this section, some fundamental notions, which are closely related to the article, are recalled.

Throughout the article ${}_3(w^F)$, ${}_3(\ell_\infty^F)$, ${}_3(c^F)$, ${}_3(c_0{}^F)$ denote the spaces of all, bounded, convergent in Pringsheim's sense, null in Pringsheim's sense fuzzy real-valued triple sequences respectively.

A triple sequence can be defined as a function $x : N \times N \times N \to R(C)$. where N, R and C denote the sets of natural and real numbers respectively.

.A fuzzy real valued triple sequence $X = \langle X_{mnl} \rangle$ is a triple infinite array of fuzzy real numbers X_{mnl} for all $m, n, l \in N$ and is denoted by $\langle X_{mnl} \rangle$ where $X_{nlk} \in R(L)$.

A fuzzy real-valued triple sequence $X = \langle X_{mnl} \rangle$ is said to be convergent in Pringsheims sense to
The fuzzy real number X, if for every $\varepsilon > 0, \exists,\ m_0 = m_0(\varepsilon), n_0 = n_0(\varepsilon), l_0 = l_0(\varepsilon) \in N$ such
That $\bar{d}(X_{mnl}, X) < \varepsilon$ for all $m \geq m_0, n \geq n_0, l \geq l_0$.

A fuzzy real-valued triple sequence $X = \langle X_{mnl} \rangle$ is said to be I_3-convergent to the fuzzy number X_0, if for all $\varepsilon > 0$, the set $\{(m,n,l) \in N \times N \times N : \bar{d}(X_{mnl}, X_0) \geq \varepsilon\} \in I_3$. We write $I_3 - \lim\ X_{mnl} = X_0$.

A fuzzy real-valued triple sequence $X = \langle X_{mnl} \rangle$ is said to be I_3-bounded if there exists a real number μ such that the set $\{(m,n,l) \in N \times N \times N : \bar{d},(X_{mnl}, \bar{0}) > \mu\} \in I_3$..

Lacunary triple sequence

A triple sequence $\theta_{r,s,p} = \{(m_r, n_s, l_p)\}(r, s, p = 0, 1, 2, ...)$ of positive integers is said to be lacunary if there exists three increasing sequences of integers $\{m_r\}, \{n_s\}, \{l_p\}$ such that

$m_0 = 0, h_r = m_r - m_{r-1} \to \infty\ as\ r \to \infty$

$n_0 = 0, h_r = n_r - n_{r-1} \to \infty\ as\ r \to \infty$

$l_0 = 0, h_r = l_r - l_{r-1} \to \infty\ as\ r \to \infty.$

Let us denote $m_{r,s,p} = m_r n_s l_p$ and $h_{r,s,p} = h_r h_s h_p$ and the intervals are determined by $\theta_{r,s,p}$ and it will be defined by

$J_{r,s,p} = \{(m,n,l): m_{r-1} < m \leq m_r, n_{r-1} < n \leq n_r, l_{p-1} < l \leq l_p\}$ and $q_r = \frac{m_r}{m_{r-1}},\ q_s = \frac{n_r}{n_{r-1}},\ q_p = \frac{l_p}{l_{p-1}}.$

A triple sequence $\langle x_{mnl} \rangle$ is said to be $\theta_{r,s,p}$ convergent to L if for every $\varepsilon > 0$ and there exists
Integers $n_0 \in N$ such that

$$\frac{1}{h_{r,s,p}} \sum_{(m,n,p) \in J_{r,s,p}} \bar{d}(x_{mnl}, L) < \varepsilon\ \forall\ r, s, p \geq n_o$$

$$\therefore \theta_{r,s,p} - \lim\ x_{mnl} = L.$$

Lacunary ideal convergence of fuzzy triple sequences:

Let $\theta_{r,s,p} = \{m_{r,s,p}\}$ be a triple lacunary sequence. Then a triple sequence $\langle X_{mnl} \rangle$ of fuzzy real numbers is said to be lacunary $I_{\theta_{r,s,p}}$-convergent to a fuzzy real numbers L if for every $\varepsilon > 0$, such that

$$\left\{(r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} \bar{d}(X_{mnl}, L) \geq \varepsilon\right\} \in I_3.$$

We write $I_{\theta_{r,s,p}} - \lim X_{mnl} = L.$

A triple sequence $\langle X_{mnl} \rangle$ of fuzzy real numbers is said to be lacunary $I_{\theta_{r,s,p}}$-null if for every $\varepsilon > 0$, such that

$$\left\{(r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} \bar{d}(X_{mnl}, \bar{0}) \geq \varepsilon\right\} \in I_3.$$

We write $I_{\theta_{r,s,p}} - \lim X_{mnl} = \bar{0}.$

Let I_3 be an admissible ideal of $N \times N \times N$. A triple sequence $\langle X_{mnl} \rangle$ is said to be $I_{\theta_{r,s,p}}$ - Cauchy if there exists a subsequence $\langle X_{m'(r)n'(s)l'(p)} \rangle$ of $\langle X_{mnl} \rangle$ such that $(m'(r), n'(s), l'(p)) \in J_{r,s,p}$ for each r , s , p $\lim_{(r,s,p)\to(\infty,\infty,\infty)} X_{m'(r)n'(s)l'(p)} = L$ and for every $\varepsilon > 0$ such that

$$\left\{(r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(X_{mnl}, X_{m'(r)n'(s)l'(p)} \right) \geq \varepsilon \right\} \in I_3.$$

Lacunary Ideal limit point and cluster point on triple sequence

Definition. Let x = $\langle x_{mnl} \rangle$ be a triple sequence. Then

An element x_0 is said to be $I_{\theta_{r,s,p}}$ limit point of x = $\langle x_{mnl} \rangle$ if there is a set $M = \{ (m_1, n_1, l_1) < (m_2, n_2 . l_2) < \text{..........} \} \in N \times N \times N$ such that the set $M' = \{ (r, s, p) \in N \times N \times N : (m_r, n_s, l_p) \in J_{r,s,p} \} \notin I_3$.

And $\theta_{r,s,p} - \lim x_{mnl} = x_0$.

An element x_0 is said to be $I_{\theta_{r,s,p}}$ - cluster point of x = $\langle x_{mnl} \rangle$ if for every $\varepsilon > 0$, we have

$$\left\{(r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(x_{mnl}, L \right) \geq \varepsilon \right\} \notin I_3$$

Let $\Lambda^I_{r,s,p}(x)$ denote the set of all $I_{\theta_{r,s,p}}$ - limit point and $\Gamma^I_{r,s,p}(x)$ denote the set of all $I_{\theta_{r,s,p}}$ -cluster points respectively.

III. MAIN RESULTS

Theorem. 3.1 Let x = $\langle x_{mnl} \rangle$ be a triple sequence. Then $\Lambda^I_{r,s,p}(x) \subset \Gamma^I_{r,s,p}(x)$.

Proof. Let $x_0 \in \Lambda^I_{r,s,p}(x)$ be any element. Then there exists a set $M \subset N \times N \times N$ Such that $M' \notin I$ where M and M' are in the above definition .

$\theta_{r,s,p} - \lim x_{mnl} = x_0$. Then for every $\varepsilon > 0$ then there exists $m_0, n_0, l_0 \in N$ such

$$\frac{1}{h_{r,s,p}} \sum_{(m,n,p)\in J_{r,s,p}} \bar{d}\left(x_{mnl}, L \right) < \varepsilon \ \forall\, r \geq m_0, s \geq n_0, p \geq l_o$$

. Therefore

$$A = \left\{(r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(x_{mnl}, L \right) < \varepsilon \right\}$$

$$\supseteq M' \setminus \{ (m_1, n_1, l_1), (m_2, n_2, l_2), \ldots\ldots (m_{m_0}, n_{n_0}, l_{l_9}) \}$$

Since I is admissible, we must have

$\supseteq M^{/} \setminus \{ (m_1 , n_1 , l_1), (m_2 , n_2 , l_2), ..., (m_{m_0} , n_{n_0} , l_{l_9}) \} \notin I_3$

$\therefore A \notin I_3$

Hence $x_0 \in \Gamma^{I}_{r,s,p}(x)$.

Theorem 3.2. Let x = $\langle x_{mnl} \rangle$ be a triple sequence, Then the following statements are equivalent

x_0 is a $I_{\theta_{r,s,p}}$ limit point of x.

There exist two triple sequences $\langle y_{mnl} \rangle$ and $\langle z_{mnl} \rangle$ such that x = y + z and $\theta_{r,s,p} - \lim y_{mnl} = x_0$ and $\{ (r,s,p) \in N \times N \times N : (m , n , l) \in J_{r,s,p} \ z_{mnl} \equiv \bar{0} \} \in I_3$

Proof. (i) ⇒ (ii)

Let (i) holds.

Then there is a set $M = \{ (m_1 , n_1 , l_1) < (m_2 , n_2 . l_2) < .... \} \in N \times N \times N$ such that the set $M^{/} = \{ (r , s , p) \in N \times N \times N : (m_r , n_s , l_p) \in J_{r,s,p} \} \notin I_3$.

And $\theta_{r,s,p} - \lim x_{mnl} = x_0$.

Let us define $\langle y_{mnl} \rangle$ and $\langle z_{mnl} \rangle$ as

$y_{mnl} = \begin{cases} x_{mnl}, & if\ (m,n,\ l) \in J_{r,s,p}: (r,s,p) \in M^{/} \\ x_0 & ,otherwise \end{cases}$ and $z_{mnl} = \begin{cases} \bar{0}, & if\ (m,n,\ l) \in J_{r,s,p}: (r,s,p) \in M^{/} \\ x_{mnl} - x_0 & otherwise \end{cases}$

If we consider $(m , n , l) \in J_{r,s,p}$ such that $(r , s , p) \in N \times N \times N - M^{/}$. Then for each $\varepsilon > 0$, we have

$$\frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} | y_{mnl} - x_0 | < \varepsilon \ \forall \ r,s,p \geq n_0$$

Hence $\theta_{r,s,p} - \lim y_{mnl} = x_0$

Now $\{ (r,s,p) \in N \times N \times N : (m , n , l) \in J_{r,s,p} , z_{mnl} \equiv \bar{0} \} \subset N \times N \times N - M^{/} \in I_3$

Therefore (i) $\Longrightarrow$ (ii)

(ii) $\Longrightarrow$ (i)

Let (ii) holds. Let $M^{/} = \{ (r,s,p) \in N \times N \times N : (m , n , l) \in J_{r,s,p} \ z_{mnl} \equiv \bar{0} \} \notin I_3$

Therefore $M^{/} \in F(I)$ and so it is an infinite set.

Lt us construct the set $M = \{ (m_1 , n_1 , l_1) < (m_2 , n_2 . l_2) < .... \} \in N \times N \times N$ such that $m_r , n_s , l_p \in J_{r,s,p}$ and $z_{m_r n_s l_p} \neq \bar{0}$. Since $x_{m_r n_s l_p} = y_{m_r n_s l_p}$ and

$\theta_{r,s,p} - \lim y_{mnl} = x_0$

Hence $\theta_{r,s,p} - \lim x_{mnl} = x_0$. This complete the proof.

Theorem 3.3. Let x = $\langle x_{mnl} \rangle$ be a triple sequence. Let I be a non-trivial admissible ideal in $N \times N \times N$. If there is a $I_{\theta_{r,s,p}}$ convergent triple sequence $y = \langle y_{mnl} \rangle$ such that $\{(m, n, p) \in N \times N \times N : y_{mnl} \neq x_{mnl}\} \in I_3$. Then x is also $I_{\theta_{r,s,p}}$ convergent.

Proof. Let $\{(m, n, p) \in N \times N \times N : y_{mnl} \neq x_{mnl}\} \in I$ and $I_{\theta_{r,s,p}} - \lim y_{mnl} = L$. Then

For every $\varepsilon > 0$, the set

$$C = \left\{(r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} \bar{d}\left(y_{mnl}, L\right) \geq \varepsilon\right\} \in I_3$$

Therefore for every $\varepsilon > 0$, we have

$$\left\{(r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} \bar{d}\left(x_{mnl}, L\right) \geq \varepsilon\right\} \subseteq$$

$$\{(m, n, p) \in N \times N \times N : y_{mnl} \neq x_{mnl}\} \cup$$

$$\left\{(r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} \bar{d}\left(y_{mnl}, L\right) \geq \varepsilon\right\} \in I_3$$

$$\left\{(r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} \bar{d}\left(x_{mnl}, L\right) \geq \varepsilon\right\} \in I_3$$

Therefore

$$\left\{(r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} | x_{mnl} - L | \geq \varepsilon\right\} \in I_3$$

x_0 is a $I_{\theta_{r,s,p}}$ limit point of x.

IV. CONCLUSION

Convergence theory is used as a basic tool in, measure spaces, sequences of random variables, information theory etc. We have introduced the notion of lacunary I-convergent multiple sequences of fuzzy real numbers having multiplicity greater than two. The relation between lacunary I-convergent and lacunary I-Cauchy triple sequences is obtained. Also some algebraic and topological properties are studied and some inclusion results are derived. The introduced notion can be applied for further investigations from different aspects.

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