Ml Estimation of Mean Time between Failrue for Two Component System in Presence of CCFS Follows Weibull Law

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Abstract- This study intended to assess the maximum likelihood estimation method for the two component repairable system . The system is assumed to be under the influence of common-cause failures (CCFs). The CCFs and individual failures follows weibull law with occurrence of chance .Numerical evidences are provided to justify the use of M L estimation procedure in the cause of system Frequency Failure functions.
Keyword: MTBF, series, parallel system, CCFS failure, MLE.

I. INTRODUCTION

Reliability analysis vary powerful tool in industrial, electrical, electronics and nuclear power plants. In 1971 have identified common cause failures (CCFs) was identified which is the event of harmonized failure of components of a system due to external causes instead of outage of components themselves . CCFs greatly reduce the reliability indices under its influence. Meachum and Atwood [1983] used BFR model for CCFS in the area of nuclear power plants. The Quantification and estimation of CCFs rates were discussed by them. Chari [1988] and Chari and U S Manyam [2003] have studied the concept of CCFs to arrive at the expression of Reliability indices like Reliability R(t), meantime between failure E(t) and Frequency Failure F(t) functions using Monrovian approach. This paper attempts the estimation meantime between failure E(t) for parallel and series system in the context of Common Cause Failures.

II. ASSUMPTIONS

- The system has two components, which are stochastically independent.
- The system is affected by individual as well as common cause failures.
- The components in the system will fail singly at the constant rate βa and failure probability is P1
- The components may fail due to common causes at the constant rate βc and with failure probability is P2 such that P1 +P2 = 1.
- Time occurrences of CCS failures and individual failures follow Weibull law.
- The individual failures and CCS failures occurring independent of each other.
- The failed components are serviced singly and service time follows exponential distribution with rate of service.

III. NOTATIONS

- βI : Individual failure rate.
- βc : Common cause failure rate.
- ES(t) : Expected time of failure for series system (MTTF / MTBF)
- $\hat{E}$S(t) : ML Estimate of Expected mean time of failure for series system.
- Ep(t) : Expected time of failure for parallel system (MTTF / MTBF)
- $\hat{E}$p(t) : M L Estimate of Expected mean time of failure for parallel
- μ:Service rate of individual components
- $\hat{\beta}_I$:sample estimation of individual failure rate
- $\hat{\beta}_c$:sample estimation of common cause failure rate
- $\hat{\mu}$: Sample estimate of service time of the components
- n = Sample size.

- N = Number of simulated samples

IV. MODEL

The assumptions of Markova model can be to drive and be formulated the Reliability function R(t) under the influence of individual as well as CCF.

The quantities β_0, β_1, β_2 are as follows

$\beta_0 = \beta_I P_1$, $\beta_1 = 2\beta_I P_1$ $\beta_2 = \beta_c P_2$, $\mu_1 = \mu$, $\mu_2 = 2\mu$,

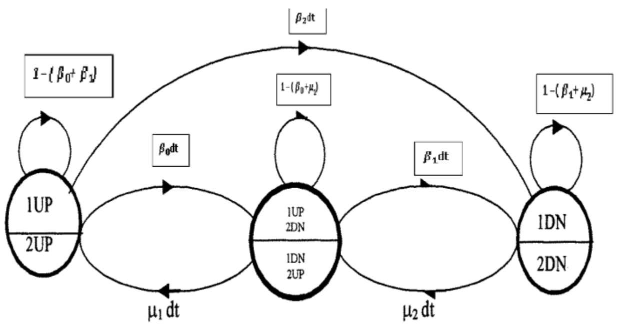

Fig. 3.1: Markov Graph For Two Component System With Individual and Common Cause Failures.

From the Markov graph the equations were formed and the probabilities of the various state of the systems i.e. Po(t), P1 (t), P2(t) are derived (see Chari [1991]).

V. TWO COMPONENT IDENTICAL SYSTEM: MEAN TIME BETWEEN FAILURES

The Mean Time Between failure function for series and parallel systems are derived using the probabilities mentioned in the section

(a) Series System

The expression of Mean time between failure function for series system is give by

$$E_s(t) = \frac{1}{(\beta_1+\beta_2)} \tag{1}$$

Where

$\beta_0 = \beta_I P_1$, $\beta_1 = 2\beta_I P_1$ $\beta_2 = \beta_c P_2$, $\mu_1 = \mu$, $\mu_2 = 2\mu$,

(b) Parallel System:

The expression of Mean time between failure function for parallel system given by

$$E_p(t) = \frac{(3\beta_0+\mu_0)}{(2\beta_0^2 + \beta_0\beta_2+\mu_0\beta_2)} \tag{2}$$

Where

$\beta_0 = \beta_I P_1$, $\beta_1 = 2\beta_I P_1$ $\beta_2 = \beta_c P_2$ $\mu_0 = \mu$, $\mu_1 = 2\mu$,

The β_I, β_c & μ are individual, Common cause failure rates and repair rate respectively and P1& P2 are the occurrence of probability of individual and CFS failure events.

VI. ESTIMATION OF RELIABILITY FUNCTION-ML ESTIMATION APPROACH

This section discusses the Maximum likelihood estimation approach for estimating Reliability function of two component parallel and series systems, which is under the influence of Individual as well as common cause failures.

Let X1, X2, X3...Xn,, be a sample of 'n' number of times between individual failures which will obey weibul law.

Let Y1 Y2, Y3 ...Yn,, be a sample of 'n number of times between common cause system failures assume to follow Weibul law.

Let Z1 Z2, Z3 ...Zn,, be a sample of 'n` number of times between service of the component assume to follows exponential law.

$$\frac{\sum_{i=1}^{n} x^{k} \ln x_i}{\sum_{i=1}^{n} x^{k}} - \frac{1}{k} - \frac{\sum_{i=1}^{n} \ln x}{n} = 0 \qquad \frac{\sum_{i=1}^{n} y^{k} \ln y_i}{\sum_{i=1}^{n} y^{k}} - \frac{1}{k} - \frac{\sum_{i=1}^{n} \ln yi}{n} = 0$$

$$\hat{\mu} = \frac{1}{\frac{\sum z_i}{n}} \quad \hat{\beta}_I = \frac{\sum_{i=1}^{n} x_i^{K}}{n} \quad \hat{\beta}_c = \frac{\sum_{i=1}^{n} y_i^{K}}{n} \tag{3}$$

are sample estimates of rate of the individual failure rate ($\hat{\beta}_I$) ,common cause failure rate ($\hat{\beta}_c$) and service rate $\hat{\mu}$ of the components respectively. under precedence of individual as well as CCS failures.

VII. ESTIMATION OF MEAN TIME BETWEEN FAILURES TWO COMPONENT SYSTEM-M L ESTIMATION APPROACH

The Mean Time Between failure function of two component identical series and parallel systems by using ML approach as follows

(a) Series System

The expression of Mean time between failure function for series system is give by

$$\widehat{EE}_s(t) = \frac{1}{(\hat{\beta}_1 + \hat{\beta}_2)} \tag{4}$$

Where

$$\hat{\beta}_0 = \hat{\beta}_I \text{ P1} \quad \hat{\beta}_1 = 2\hat{\beta}_I \text{ P1} \quad \hat{\beta}_2 = \hat{\beta}_c \text{ P2} \tag{5}$$

$\hat{\beta}_I$, $\hat{\beta}_c$, $\hat{\mu}$ P1&P2 are individual failures rate, Common cause failure rate, repair rate and probability of occurrence of individual as well as CCS failure

(b) Parallel System:

The expression of Mean time between failure function for parallel system given by

$$\widehat{EE}_p(t) = \frac{(3\hat{\beta}_0 + \hat{\mu}_0)}{(2\hat{\beta}_1^2 + \hat{\beta}_0\hat{\beta}_2 + \hat{\mu}_0\hat{\beta}_2)} \tag{6}$$

Where

$$\hat{\beta}_0 = \hat{\beta}_I \text{ P1} \quad \hat{\beta}_1 = 2\hat{\beta}_I \text{ P1} \quad \hat{\beta}_2 = \hat{\beta}_c \text{ P2}$$

$\hat{\beta}_I$, $\hat{\beta}_c$, $\hat{\mu}$ P1&P2 are individual failures rate, Common cause failure rate, repair rate and probability of occurrence of individual as well as CCS failure

VIII. SIMULATION AND VALIDITY

For a range of specified values of the rates of individual (βI), common cause failures(βc) and service rates(μ) and for the samples of sizes n = 5 (5) 30 are simulated using computer package developed in this paper and the sample estimates are computed for N = 10000 (10000) 100000 and mean square error (MSE) of the estimates for Es(t) , EP(t) , were obtained and given in tables [Tab.1 Tab.2,] The tables and graphs are seen in the. For large samples Maximum Likelihood estimators are undisputedly better since they are CAN estimators. However it is interest to note that for a sample size as low as five (n=5) also M L estimate is still seen to be reasonably good giving near accurate estimate in this case. This shows that ML method of estimator is quite useful in this context.

Table-1:Results of the simulations for mean time between failure function of series system

$\beta_I = 0.02 \quad \beta_c = 0.03 \quad \mu = 1$ P1= 0.02 k =1

Sample size = 5					Sample size = 10				
N	Es(t)	$\hat{E}_s(t)$	s.s	M.S.E	N	Es(t)	$\hat{E}_s(t)$	s.s	M.S.E
10000	0.159363	0.019544	0.000002	0.001398	10000	0.159363	0.015794	0.000002	0.001436
20000	0.159363	0.019636	0.000001	0.000988	20000	0.159363	0.007913	0.000001	0.001071
30000	0.159363	0.014709	0.000001	0.000835	30000	0.159363	0.021304	0.000001	0.000797
40000	0.159363	0.020019	0	0.000697	40000	0.159363	0.020344	0	0.000695
50000	0.159363	0.036548	0	0.000549	50000	0.159363	0.025846	0	0.000597
60000	0.159363	0.02256	0	0.000558	60000	0.159363	0.007583	0	0.00062
70000	0.159363	0.032968	0	0.000478	70000	0.159363	0.010111	0	0.000564
80000	0.159363	0.015977	0	0.000507	80000	0.159363	0.022706	0	0.000483
90000	0.159363	0.016698	0	0.000476	90000	0.159363	0.013572	0	0.000486
100000	0.159363	0.003144	0	0.000494	100000	0.159363	0.016965	0	0.00045

Sample size = 15	Sample size = 20

N	Es(t)	$\hat{E}_s(t)$	s.s	M.S.E	N	Es(t)	$\hat{E}_s(t)$	s.s	M.S.E
10000	0.159363	0.025349	0.000002	0.00134	10000	0.159363	0.01974	0.000002	0.001396
20000	0.159363	0.016939	0.000001	0.001007	20000	0.159363	0.017712	0.000001	0.001002
30000	0.159363	0.011997	0.000001	0.000851	30000	0.159363	0.016228	0.000001	0.000826
40000	0.159363	0.020434	0	0.000695	40000	0.159363	0.017828	0.000001	0.000708
50000	0.159363	0.023056	0	0.00061	50000	0.159363	0.014104	0	0.00065
60000	0.159363	0.023056	0	0.000556	60000	0.159363	0.018281	0	0.000576
70000	0.159363	0.017574	0	0.000536	70000	0.159363	0.018776	0	0.000531
80000	0.159363	0.012545	0	0.000519	80000	0.159363	0.025089	0	0.000475
90000	0.159363	0.022822	0	0.000455	90000	0.159363	0.018472	0	0.00047
100000	0.159363	0.019933	0	0.000441	100000	0.159363	0.024878	0	0.000425

Sample size = 25					Sample size = 30				
N	Es(t)	$\hat{E}_s(t)$	s.s	M.S.E	N	Es(t)	$\hat{E}_s(t)$	s.s	M.S.E
10000	0.159363	0.018757	0.000002	0.001406	10000	0.159363	0.014818	0.000002	0.001445
20000	0.159363	0.019507	0.000001	0.000989	20000	0.159363	0.022854	0.000001	0.000965
30000	0.159363	0.024769	0.000001	0.000777	30000	0.159363	0.014789	0.000001	0.000835
40000	0.159363	0.018728	0	0.000703	40000	0.159363	0.019564	0	0.000699
50000	0.159363	0.016279	0	0.00064	50000	0.159363	0.023705	0	0.000607
60000	0.159363	0.016141	0	0.000585	60000	0.159363	0.022238	0	0.00056
70000	0.159363	0.018957	0	0.000531	70000	0.159363	0.019886	0	0.000527
80000	0.159363	0.019901	0	0.000493	80000	0.159363	0.013846	0	0.000514
90000	0.159363	0.022523	0	0.000456	90000	0.159363	0.016448	0	0.000476
100000	0.159363	0.021526	0	0.000436	100000	0.159363	0.019208	0	0.000443

Table-2:Results of the simulations for mean time between failure function of parallel system
$\beta_1 = 0.2$ $\beta_c = 0.3$ $\mu = 1$ P1= 0.5 , k =1

Sample size = 5					Sample size = 10				
N	Ep(t)	$\hat{E}_p(t)$	s.s	M.S.E	N	Ep(t)	$\hat{E}_p(t)$	s.s	M.S.E
10000	0.463636	0.360799	0.000001	0.001028	10000	0.463636	0.317007	0.000002	0.001466
20000	0.463636	0.53719	0	0.00052	20000	0.463636	0.401559	0	0.000439
30000	0.463636	0.584788	0	0.000699	30000	0.463636	0.414597	0	0.000283
40000	0.463636	0.401282	0	0.000312	40000	0.463636	0.415297	0	0.000242
50000	0.463636	0.249662	0.000001	0.000957	50000	0.463636	0.420277	0	0.000194
60000	0.463636	0.338601	0	0.00051	60000	0.463636	0.391685	0	0.000294
70000	0.463636	0.543977	0	0.000304	70000	0.463636	0.700501	0.000001	0.000895
80000	0.463636	0.378333	0	0.000302	80000	0.463636	0.325342	0	0.000489
90000	0.463636	0.264525	0	0.000664	90000	0.463636	0.388911	0	0.000249
100000	0.463636	0.330954	0	0.00042	100000	0.463636	0.407134	0	0.000179

Sample size = 15					Sample size = 20				
N	Ep(t)	$\hat{E}_p(t)$	s.s	M.S.E	N	Ep(t)	$\hat{E}_p(t)$	s.s	M.S.E
10000	0.463636	0.463094	0	0.000005	10000	0.463636	0.449395	0	0.000142
20000	0.463636	0.37926	0	0.000597	20000	0.463636	0.441694	0	0.000155
30000	0.463636	0.51761	0	0.000312	30000	0.463636	0.42997	0	0.000194
40000	0.463636	0.472095	0	0.000042	40000	0.463636	0.433335	0	0.000152

50000	0.463636	0.374768	0	0.000397	50000	0.463636	0.414249	0	0.000221
60000	0.463636	0.283824	0.000001	0.000734	60000	0.463636	0.41992	0	0.000178
70000	0.463636	0.388842	0	0.000283	70000	0.463636	0.476209	0	0.000048
80000	0.463636	0.413341	0	0.000178	80000	0.463636	0.41183	0	0.000183
90000	0.463636	0.470477	0	0.000023	90000	0.463636	0.359871	0	0.000346
100000	0.463636	0.49206	0	0.00009	100000	0.463636	0.462841	0	0.000003

Sample size = 25					Sample size = 30				
N	Ep(t)	$\hat{E}_p(t)$	s.s	M.S.E	N	Ep(t)	$\hat{E}_p(t)$	s.s	M.S.E
10000	0.463636	0.337308	0.000002	0.001263	10000	0.463636	0.428195	0	0.000354
20000	0.463636	0.432081	0	0.000223	20000	0.463636	0.369433	0	0.000666
30000	0.463636	0.355813	0	0.000623	30000	0.463636	0.484534	0	0.000121
40000	0.463636	0.400125	0	0.000318	40000	0.463636	0.424541	0	0.000195
50000	0.463636	0.465453	0	0.000008	50000	0.463636	0.366801	0	0.000433
60000	0.463636	0.46117	0	0.00001	60000	0.463636	0.365837	0	0.000399
70000	0.463636	0.369901	0	0.000354	70000	0.463636	0.470786	0	0.000027
80000	0.463636	0.417573	0	0.000163	80000	0.463636	0.420837	0	0.000151
90000	0.463636	0.316841	0	0.000489	90000	0.463636	0.452785	0	0.000036
100000	0.463636	0.379529	0	0.000266	100000	0.463636	0.48008	0	0.000052

IX. CONCLUSIONS

This paper attempts to evaluate the estimate of the men time between failure function s in the presence of common cause and individual failure. The ML method proposed here is giving almost accuracy estimation in case of sample size 10 and above which is verified by the simulation in the absence analytical approach. Also these results suggested the ML estimate is reasonable good and gives accurate estimates even for sample size n=5 therefore this paper identifies the use of thee an ML method of estimator justified through empirical means estimation of the mean time between failure of two component system in presence of CCFs as well as individual failures.

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