On Comparison Of Topologies

R. Chitralekha[1], M. Anitha[2], N. Meena[3]
[1]*Department of Mathematics, Rani Anna Govt. College, Tirunelveli-8, Tamilnadu, India.*
[2]*Department of Mathematics, Rani Anna Govt. College, Tirunelveli-8, Tamilnadu, India.*
[3]*Department of Mathematics, M.D.T.Hindu College, Tirunelveli, Tamilnadu, India.*
Affiliated to ManonmaniamSundaranar University,Abishekapatti, Tirunelveli-12, India

Abstract- Weston introduced the concept of coupled relations in the year 1957 on the family topologies on a nonempty set. Dvalishvili compiled the results of this relation in 2005. In 2009, these relations were further investigated by Thamizharasi and Thangavelu. In this paper, new coupled relations are defined and some of the concepts in topology are studied by using these coupled relations.
Key words: Bitopology, coupled relations, semiopen set, preopen set. MSC 2010: 54A10, 54E55.

I. INTRODUCTION AND PRELIMINARIES

Norman Levine, Njastad, Mashouret.al.,Andrijevic, Indira et.al. andUshaparameswari et.al. introduced and studied respectively the concepts of semiopen[5], α-open[7], preopen[6], β-open[1],b-open[2], *b-open[3]and b#-open[10] sets in topology. The concepts of coupled relations in bitopology in 2005 were discussed in Dvalishvili[3] and Weston[11]. These relations were further investigated by Thamizharasi and Thangavelu [10]. In this paper, above nearly open sets in topological and in bitopological settings are characterized by using the coupled relations. Throughout this paper $(X,\tau1,\tau2)$ is a bitopological space; A and B are subsets of X, CliA and IntiA denote the closure of interior of A in $(X,\tau i)$; the members of τi are called the i-open sets in $(X,\tau1,\tau2)$ and $i\neq j$. The phrase " iff " denotes " if and only if ".

Definition 1.1: τi is coupled to τj [11] if $CliA\subseteq CljA$ for every i-open set A in $(X,\tau1,\tau2)$.

Definition 1.2: The bitopology $(\tau1,\tau2)$ is coupled[11] on X if $\tau1$ is coupled to $\tau2$ and $\tau2$ is coupled to $\tau1$.

II. WEAK COUPLED RELATIONS

Definition 2.1:

(i).τi is α-coupled to τj if $CliA\subseteq CljA$ for every i-α-open set A .
(ii). τi is semicoupled to τj if$CliA\subseteq CljA$ for every i-semiopen set A .
(iii).τi is precoupled to τj if $CliA\subseteq CljA$ for every i-preopen set A .
(iv).τi is β-coupled to τj if $CliA\subseteq CljA$ for every i-β-open set A .
(v).τi is b-coupled to τj if $CliA\subseteq CljA$ for every i-b-open set A .
(vi).τi is *b-coupled to τj if $CliA\subseteq CljA$ for every i-*b-open set A .
(vii).τi is b#-coupled to τj if $CliA\subseteq CljA$ for every i- b#-open set A .

The above relations are characterized by the interior operators as stated below.

Proposition 2.2:

(i).τi is α-coupled to τjiff$IntjB\subseteq IntiB$ for every i-α-closed set B .
(ii). τi is semi coupled to τjiff$IntjB\subseteq IntiB$ for every i-semiclosed set B .
(iii).τi is pre coupled to τjiff$IntjB\subseteq IntiB$ for every i-preclosed set B .
(iv).τi is β-coupled to τjiff$IntjB\subseteq IntiB$ for every i-β-closed set B .
(v).τi is b-coupled to τjiff$IntjB\subseteq IntiB$ for every i-b-closed set B .
(vi).τi is *b-coupled to τjiff$IntjB\subseteq IntiB$ for every i-*b-closed set B.
(vii).τi is b#-coupled to τjiff$IntjB\subseteq IntiB$ for every i- b#-closed set B .

Proposition 2.3:

(i). If $(\tau1,\tau2)$is α-coupled then $CliA = CljA$ for every $A\in\alpha O(X,\tau1)\cap\alpha O(X,\tau2)$.
and$IntiB = IntjB$ for every $B\in\alpha C(X,\tau1)\cap\alpha C(X,\tau2)$.
(ii). If $(\tau1,\tau2)$is semicoupled then $CliA = CljA$ for every $A\in SO(X,\tau1)\cap SO(X,\tau2)$ and
$IntiB=IntjB$ for every $B\in SC(X,\tau1)\cap SC(X,\tau2)$.
(iii). If $(\tau1,\tau2)$ is precoupled then $CliA = CljA$ for every $A\in PO(X,\tau1)\cap PO(X,\tau2)$ and $IntiB = IntjB$
for every $B\in PC(X,\tau1)\cap PC(X,\tau2)$.

(iv). If $(\tau1,\tau2)$is β-coupled then CliA =CljA for every $A\in\beta O(X,\tau1)\cap\beta O(X,\tau2)$ and IntiB =IntjB for every $B\in\beta C(X,\tau1)\cap\beta C(X,\tau2)$.
(v). If $(\tau1,\tau2)$is b-coupled then CliA =CljA for every $A\in bO(X,\tau1)\cap bO(X,\tau2)$ and IntiB =IntjB for every $B\in bC(X,\tau1)\cap bC(X,\tau2)$.
(vi). If $(\tau1,\tau2)$is *b-coupled then CliA =CljA for every $A\in *bO(X,\tau1)\cap *bO(X,\tau2)$ and IntiB = IntjB for every $B\in *bC(X,\tau1)\cap *bC(X,\tau2)$.
(vii). If $(\tau1,\tau2)$ is b#-coupled then CliA =CljA for every $A\in b\#O(X,\tau1)\cap b\#O(X,\tau2)$ and IntiB = IntjB for every $B\in b\#C(X,\tau1)\cap b\#C(X,\tau2)$.

III. STRONG COUPLED RELATIONS

Definition 3.1:
(i).τi is r-coupled to τj if $CliA\subseteq CljA$ for every i-regular open set A .
(ii).τi is δ-coupled to τj if $CliA\subseteq CljA$ for every i-δ-open set A .
(iii).τi is θ-coupled to τj if $CliA\subseteq CljA$ for every i-θ-open set A.

Proposition 3.2:
(i).τi is r-coupled to τjiff$IntjB\subseteq IntiB$ for every i-regular closed set B .
(ii).τi is δ-coupled to τjiff$IntjB\subseteq IntiB$ for every i-δ-closed set B.
(iii).τi is θ-coupled to τjiff$IntjB\subseteq IntiB$ for every i-θ-closed set B .

Proposition 3.3:
(i). If $(\tau1,\tau2)$is r-coupled then CliA =CljA for every $A\in RO(X,\tau1)\cap RO(X,\tau2)$.
andIntiB =IntjB for every $B\in RC(X,\tau1)\cap RC(X,\tau2)$.
(ii). If $(\tau1,\tau2)$is δ-coupled then CliA =CljA for every $A\in\delta O(X,\tau1)\cap\delta O(X,\tau2)$ and IntiB =IntjB for every $B\in\delta C(X,\tau1)\cap\delta C(X,\tau2)$.
(iii). If $(\tau1,\tau2)$is θ-coupled then CliA =CljA for every $A\in\theta O(X,\tau1)\cap\theta O(X,\tau2)$ and IntiB =IntjB for every $B\in\theta C(X,\tau1)\cap\theta C(X,\tau2)$.

IV. REFERENCES

[1] M.E.Abd El-Monsef S.N. El-Deeb, and R.A.Mohmoud, "β-open sets and β-continuous mappings", Bull. Fac. Sci. Assiut Univ., vol.12, pp.77-90, 1983.
[2] D.Andrijevic , "On b-open sets", Mat.Vestnik, vol.18, pp.59-64, 1996.
[3] Dvalishvili, "Bitopologicalspaces:Theory Relations with Generalized Algebraic Structures and Applications", North-Holland Mathematics studies Elsevier Science B.V Anasterdam Vol.199, 2005.
[4] T.Indira and K.Rekha, "Decomposition of Continuity via *b-open set", ActaCiencia Indica(M), vol.39, no,1, pp.73-85, 2013.
[5] N.Levine , "Semi-open sets and semi continuity in topological spaces", Amer. Math. Monthly, vol.70, pp.36-41, 1963.
[6] A.S.Mashhour ,M.E.Abd El-Monsef and S.N.El-Deeb , "On pre-continuous and weak pre-continuous functions", Proc. Math. Phys. Soc. Egypt, vol.53, pp.47-53, 1982.
[7] O.Njastad , "Some classes of nearly open sets", Pacific J.Math., vol.15, 961-970, 1965.
[8] M.H.Stone , "Application of the theory of Boolean rings to the general topology", Trans. A.M.S. vol.41, pp.375-481, 1937.
[9] G.Thamizharasi and P.Thangavelu, " Studies in Bitopological Spaces", Ph.D, Thesis, ManonmaniamSundaranar University, Tirunelveli-12, India, 2010.
[10] R.UshaParameswari and P.Thangavelu, "On b#-open sets", International Journal of Mathematics Trends and Technology, vol.3, no.5, pp.202 – 218, 2014.
[11] J.D. Weston , " On the comparison of topologies", J.London Math. Soc. vol.32, pp. 342-354, 1957.