On Comparison Of Topologies

R. Chitralekha¹, M. Anitha², N. Meena³

¹Department of Mathematics, Rani Anna Govt. College, Tirunelveli-8, Tamilnadu, India.
²Department of Mathematics, Rani Anna Govt. College, Tirunelveli-8, Tamilnadu, India.
³Department of Mathematics, M.D.T.Hindu College, Tirunelveli, Tamilnadu, India.
Affiliated to ManonmaniamSundaranar University, Abishekappatti, Tirunelveli-12, India

Abstract- Weston introduced the concept of coupled relations in the year 1957 on the family topologies on a nonempty set. Dvalishvili compiled the results of this relation in 2005. In 2009, these relations were further investigated by Thamizharasi and Thangavelu. In this paper, new coupled relations are defined and some of the concepts in topology are studied by using these coupled relations.

Key words: Bitopology, coupled relations, semiopen set, preopen set. MSC 2010: 54A10, 54E55.

I. INTRODUCTION AND PRELIMINARIES

Norman Levine, Njastad, Mashoure et al., Andrijevic, Indira et al. and Ushaparameswari et al. introduced and studied respectively the concepts of semiopen[5], α-open[7], preopen[6], β-open[1], b-open[2], *b-open[3] and b#-open[10] sets in topology. The concepts of coupled relations in bitopology in 2005 were discussed in Dvalishvili[3] and Weston[11]. These relations were further investigated by Thamizharasi and Thangavelu [10]. In this paper, above nearly open sets in topological and in bitopological settings are characterized by using the coupled relations. Throughout this paper (X, \( \tau_1, \tau_2 \)) is a bitopological space; A and B are subsets of X. Cl(\( A \)) and Int(\( A \)) denote the closure of interior of A in \( (X, \tau_1) \); the members of \( \tau_i \) are called the i-open sets in \( (X, \tau_1, \tau_2) \) and \( i \neq j \). The phrase “iff” denotes “if and only if”.

Definition 1.1: \( \tau_i \) is coupled to \( \tau_j \) [11] if Cl(A) \( \subseteq \) Cl(\( A \)) for every i-open set \( A \) in \( (X, \tau_1, \tau_2) \).

Definition 1.2: The bitopology \( (\tau_1, \tau_2) \) is coupled[11] on X if \( \tau_1 \) is coupled to \( \tau_2 \) and \( \tau_2 \) is coupled to \( \tau_1 \).

II. WEAK COUPLED RELATIONS

Definition 2.1:
(i). \( \tau_1 \) is \( \alpha \)-coupled to \( \tau_j \) if Cl(A) \( \subseteq \) Cl(\( A \)) for every i-\( \alpha \)-open set \( A \).
(ii). \( \tau_1 \) is semi coupled to \( \tau_j \) if Cl(A) \( \subseteq \) Cl(\( A \)) for every i-semiopen set \( A \).
(iii). \( \tau_1 \) is precoupled to \( \tau_j \) if Cl(A) \( \subseteq \) Cl(\( A \)) for every i-preopen set \( A \).
(iv). \( \tau_1 \) is \( \beta \)-coupled to \( \tau_j \) if Cl(A) \( \subseteq \) Cl(\( A \)) for every i-\( \beta \)-open set \( A \).
(v). \( \tau_1 \) is b-coupled to \( \tau_j \) if Cl(A) \( \subseteq \) Cl(\( A \)) for every i-b-open set \( A \).
(vi). \( \tau_1 \) is *b-coupled to \( \tau_j \) if Cl(A) \( \subseteq \) Cl(\( A \)) for every i-*b-open set \( A \).
(vii). \( \tau_1 \) is b#-coupled to \( \tau_j \) if Cl(A) \( \subseteq \) Cl(\( A \)) for every i-b#-open set \( A \).

The above relations are characterized by the interior operators as stated below.

Proposition 2.2:
(i). \( \tau_1 \) is \( \alpha \)-coupled to \( \tau_j \) if Cl(\( i \) Int(\( j \)) Int(\( B \)) \( \subseteq \) Int(\( B \)) for every i-\( \alpha \)-closed set \( B \).
(ii). \( \tau_1 \) is semi coupled to \( \tau_j \) if Cl(\( i \) Int(\( j \)) Int(\( B \)) \( \subseteq \) Int(\( B \)) for every i-semiclosed set \( B \).
(iii). \( \tau_1 \) is pre coupled to \( \tau_j \) if Cl(\( i \) Int(\( j \)) Int(\( B \)) \( \subseteq \) Int(\( B \)) for every i-preclosed set \( B \).
(iv). \( \tau_1 \) is \( \beta \)-coupled to \( \tau_j \) if Cl(\( i \) Int(\( j \)) Int(\( B \)) \( \subseteq \) Int(\( B \)) for every i-\( \beta \)-closed set \( B \).
(v). \( \tau_1 \) is b-coupled to \( \tau_j \) if Cl(\( i \) Int(\( j \)) Int(\( B \)) \( \subseteq \) Int(\( B \)) for every i-b-closed set \( B \).
(vi). \( \tau_1 \) is *b-coupled to \( \tau_j \) if Cl(\( i \) Int(\( j \)) Int(\( B \)) \( \subseteq \) Int(\( B \)) for every i-*b-closed set \( B \).
(vii). \( \tau_1 \) is b#-coupled to \( \tau_j \) if Cl(\( i \) Int(\( j \)) Int(\( B \)) \( \subseteq \) Int(\( B \)) for every i-b#-closed set \( B \).

Proposition 2.3:
(i). If \( \tau_1, \tau_2 \) is \( \alpha \)-coupled then Cl(\( i \) A) = Cl(\( j \) A) for every A \( \in \alpha \)O(\( X, \tau_1 \)) \( \cap \) \( \alpha \)O(\( X, \tau_2 \)) and Int(\( i \) B) = Int(\( j \) B) for every B \( \in \alpha \)C(\( X, \tau_1 \)) \( \cap \) \( \alpha \)C(\( X, \tau_2 \)).
(ii). If \( \tau_1, \tau_2 \) is semi coupled then Cl(\( i \) A) = Cl(\( j \) A) for every A \( \in \) SO(\( X, \tau_1 \)) \( \cap \) SO(\( X, \tau_2 \)) and Int(\( i \) B) = Int(\( j \) B) for every B \( \in \) SC(\( X, \tau_1 \)) \( \cap \) SC(\( X, \tau_2 \)).
(iii). If \( \tau_1, \tau_2 \) is precoupled then Cl(\( i \) A) = Cl(\( j \) A) for every A \( \in \) PO(\( X, \tau_1 \)) \( \cap \) PO(\( X, \tau_2 \)) and Int(\( i \) B) = Int(\( j \) B) for every B \( \in \) PC(\( X, \tau_1 \)) \( \cap \) PC(\( X, \tau_2 \)).
(iv). If $(\tau_1, \tau_2)$ is $\beta$-coupled then $\text{Cl} A = \text{Cl} j A$ for every $A \in \beta O(X, \tau_1) \cap \beta O(X, \tau_2)$ and $\text{Int} B = \text{Int} j B$ for every $B \in \beta C(X, \tau_1) \cap \beta C(X, \tau_2)$.

(v). If $(\tau_1, \tau_2)$ is $b$-coupled then $\text{Cl} A = \text{Cl} j A$ for every $A \in b O(X, \tau_1) \cap b O(X, \tau_2)$ and $\text{Int} B = \text{Int} j B$ for every $B \in \beta C(X, \tau_1) \cap \beta C(X, \tau_2)$.

(vi). If $(\tau_1, \tau_2)$ is $b^*$-coupled then $\text{Cl} A = \text{Cl} j A$ for every $A \in b^* O(X, \tau_1) \cap b^* O(X, \tau_2)$ and $\text{Int} B = \text{Int} j B$ for every $B \in \beta C(X, \tau_1) \cap \beta C(X, \tau_2)$.

(vii). If $(\tau_1, \tau_2)$ is $b\#$-coupled then $\text{Cl} A = \text{Cl} j A$ for every $A \in b\# O(X, \tau_1) \cap b\# O(X, \tau_2)$ and $\text{Int} B = \text{Int} j B$ for every $B \in \beta C(X, \tau_1) \cap \beta C(X, \tau_2)$.

### III. STRONG COUPLED RELATIONS

**Definition 3.1:**

(i). $\tau_i$ is $r$-coupled to $\tau_j$ if $\text{Cl} A \subseteq \text{Cl} j A$ for every $i$-regular open set $A$.

(ii). $\tau_i$ is $\delta$-coupled to $\tau_j$ if $\text{Cl} A \subseteq \text{Cl} j A$ for every $i$-$\delta$-open set $A$.

(iii). $\tau_i$ is $\theta$-coupled to $\tau_j$ if $\text{Cl} A \subseteq \text{Cl} j A$ for every $i$-$\theta$-open set $A$.

**Proposition 3.2:**

(i). $\tau_i$ is $r$-coupled to $\text{Int} j B \subseteq \text{Int} B$ for every $i$-regular closed set $B$.

(ii). $\tau_i$ is $\delta$-coupled to $\text{Int} j B \subseteq \text{Int} B$ for every $i$-$\delta$-closed set $B$.

(iii). $\tau_i$ is $\theta$-coupled to $\text{Int} j B \subseteq \text{Int} B$ for every $i$-$\theta$-closed set $B$.

**Proposition 3.3:**

(i). If $(\tau_1, \tau_2)$ is $r$-coupled then $\text{Cl} A = \text{Cl} j A$ for every $A \in RO(X, \tau_1) \cap RO(X, \tau_2)$.

(ii). If $(\tau_1, \tau_2)$ is $\delta$-coupled then $\text{Cl} A = \text{Cl} j A$ for every $A \in \delta O(X, \tau_1) \cap \delta O(X, \tau_2)$.

(iii). If $(\tau_1, \tau_2)$ is $\theta$-coupled then $\text{Cl} A = \text{Cl} j A$ for every $A \in \theta O(X, \tau_1) \cap \theta O(X, \tau_2)$.

### IV. REFERENCES


