Combinatorial properties and n-ary topology on product of power sets

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Abstract- Nithyanantha Jothi and Thangavelu studied the properties of the product of two power sets and introduced the concept of binary topology. In this paper, properties of the product of arbitrarily n-power sets are discussed where n >2. Further an n-ary topology on the product of power sets similar to binary topology is introduced and studied.
Keywords: Binary topology, n-ary sets, n-ary topology, product topology. MSC 2010: 54A05, 54A99.

I. INTRODUCTION

The concept of a binary topology was introduced and studied by Nithyanantha Jothi and Thangavelu [4-9] in 2011. Recently Lellish Thivagar et.al.[3] extended this notion to supra topology, Jamal Mustafa[2] to generalized topology and Benchalli et.al.[1] to soft topology. Nithyanantha Jothi and Thangavelu[9] extended the concepts of regular open and semiopen sets in point set topology to binary topology. The authors[10] studied the notion of nearly binary open sets in binary topological spaces. In this paper, the notion of n-ary topology is introduced and its properties are discussed. Section 1 deals with basic properties of the product of power sets and section 2 deals with n-ary topology with sufficient examples and some basic results.

II. PRODUCT OF POWER SETS

Let X1, X2, X3,…,Xn be the non empty sets. Then P(Xi) denotes the collection of all subsets of X, called the power set of Xi. P(X1)× P(X2)×…× P(Xn) is the cartesian product of the power sets P(X1), P(X2), …, P(Xn). Examples can be constructed to show that the two notions 'product of power sets' and 'power set of the products' are different. When |X1|=|X2| =2, it is noteworthy to see that |P(X1×X2)|= |P(X1)×P (X2)| =16. But this is not always true as shown in the next proposition .

2.1. Proposition-

Suppose X1,X2,…, Xn are finite non empty sets satisfying one of the conditions (i).n>2 , |Xi|>1for each i∈{1,2,…,n}; (ii).n=2, |X1|>2 ,|X2|≥2; (iii).n=2, |X2|>2, |X1| ≥2.Then |P(X1×X2×…×Xn)| > | P(X1)× P(X2)×…× P(Xn)|.

Proof: Suppose Xi has mi elements for each i∈1,2,…,n}. Then each P(Xi) has 2^{m_i} members. Therefore X1×X2×…×Xn has m1×m2×…×mn elements that implies P(X1×X2×…×Xn) has $2^{m_1\times m_2\times..\times m_n}$ elements where as P(X1)× P(X2)×…× P(Xn) has $2^{m_1}.2^{m_2}...2^{m_n}$ members. That is P(X1)× P(X2)×…× P(Xn) has $2^{m_1+m_2+...+m_n}$ members. Under the conditions on n and |Xi| it follows that $2^{m_1\times m_2\times..\times m_n} > 2^{m_1+m_2+...+m_n}$. This shows that
|P(X1×X2×…×Xn)|>| P(X1)× P(X2)×…× P(Xn)|.

2.2. Proposition-

Let N be the set of all natural numbers and a,b,c,d,e∈N.
(i). The equation ab=a+b has exactly one solution in N.
(ii).The equation abc=a+b+c has at exactly six solutions in N.

Proof: Solutions can be found by inspection method. If a=1 then ab=a+b gives b=1+b that implies (a=1, b≥1) cannot be a solution for (i). Clearly (a=2, b=1) is not a solution for (i). But (a=2, b=2) is a solution for (i). Now suppose (a,b) is a solution for (i) in N. Then ab=a+b that implies a divides a+b. Since a divides itself it follows that a divides b. Similarly b divides a that implies a=b from which it follows that a2=2a . This proves that a=b=2. Therefore (2,2) is the only solution of (i) in N. This proves (i).

Now for the equation (ii), suppose a=1,we get bc=1+b+c. 'b=1' is not possible. If b=2 then 2c=1+1+c=2+c that implies c=2. Therefore (a,b,c) = (1,2,3) is a solution for (ii). The other solutions are (1,3,2), (2,3,1), (2,1,3), (3,1,2), (3,2,1). Hence the equation abc=a+b+c has at least six solutions in N. Now suppose (a,b,c) is any solution in N for (ii). Then no two of a,b,c are equal to 1. For, suppose a=b=1. Then from (ii), c= 2+c that is not possible. No two of a,b,c are equal. For, if a=b then a2c = 2a+c that implies c divides 2a. Therefore 2a = kc for some natural number k. Then using this in a2c = 2a+c we get a2 = k+1. Therefore 2a = kc=(a2-1)c which implies c = 2a/(a2-1). Since a>1 and since a< (a2-1) we see that c<2 that implies c=1 . Then using c=1 and a=b in (ii) we get a2 = 2a+1 which has no solution for a in N. Hence we conclude that a,b,c are all distinct. We assume that a<b<c. If a=1 then bc=1+b+c that implies b=2 and c=3. If a =k>1, b≥k+1 and c ≥k+2 then abc ≥ k(k+1)(k+2)>(k+1)(k+2)>6 and a+b+c ≥ 3k+3 >6. Therefore any solution other than the permutations of (1,2,3) must satisfy a+b+c =abc >6. This shows that the equation (ii) has exactly six solutions in N.

2.3. Proposition-

For any integer k>2, the equation a1+a2+…+ak = a1.a2.….ak has at least k(k–1) solutions in N.

Proof: Let (a1,a2,…,ak) be a positive integral solution for the equation

a1+a2+…+ak = a1.a2.….ak. (Eqn.1)

Then it is not possible to have a1=a2=…=ak =1. For if a1=a2=…=ak =1 then k=1. Again if a1=a2=…=ak-1 =1 and ak>1 then k-1+ ak = ak that implies k=1.

Suppose a1=a2=…=ak-2 =1 , ak-1>1 and ak>1.Then k-2+ ak-1 +ak = ak-1.ak.

Let ak-1 =r >1. Then r. ak = k+r-2 + ak that implies

$$ak = \frac{k+r-2}{r-1} = 1+\frac{k-1}{r-1}. \qquad \text{(Eqn.2)}$$

If r>k then $\frac{k-1}{r-1}$ is a proper fraction that implies ak is not an integer. Therefore we have 2≤ r≤ k. If r=2 then ak =k and if r=k then ak =k. Therefore if

a1=a2=…=ak-2 =1 , ak-1=2 and ak=k then (a1,a2,…,ak) is a solution of (Eqn.1)

This shows that (a1,a2,…,ak) = (1,1,…1,2, k) is a solution for (Eqn.1). Clearly any permutation of (1,1,…1,2, k) is also a solution for (Eqn.1).

Therefore the number of such solutions is $\frac{k!}{(k-2)!}$ = (k–1)k. Depending upon the values of k, (Eqn.1) may have other solutions. For take k=5. Then

The equation abcde = a+b+c+d+e has at least 20 solutions in N which may be got by taking r=2 in (Eqn.2). If we put r =3 in (Eqn.2) we get e=3 that implies (1,1,1,3,3) is also a solution for abcde = a+b+c+d+e. Therefore for k>2, (Eqn.1) has at least k(k-1) solutions in N.

The following proposition can be established by choosing r in Eqn.2 such that $\frac{k-1}{r-1}$ is a positive integer.

2.4. Proposition-

(i).If k=5, 10 then (Eqn.1) has at least 3k(k-1)/2 solutions in N.

(ii).If k=7, 9,11 then (Eqn.1) has at least 2k(k-1) solutions in N.

(iii).If k=13 then (Eqn.1) has at least 3k(k-1) solutions in N.

From the above discussion, the following lemma can be easily established.

2.5 Proposition-

For any integer k>1, each of the strict in equations a1+a2+…+ak < a1.a2.….ak and a1+a2+…+ak > a1.a2.….ak has at least one solution in N.

The above discussions lead to the following proposition.

2.6. Proposition-

Let |Xi| = mi for each i∈{1,2,3,…,k}. Then |P(X1×X2×…×Xk)|= | P(X1)× P(X2)×…× P(Xn)|, P(X1×X2×…×Xk)| > | P(X1)× P(X2)×…× P(Xn)| and |P(X1×X2×…×Xk)| < | P(X1)× P(X2)×…× P(Xn)| according as (m1,m2,…,mk) is a solution of a1+a2+…+ak = a1.a2.….ak , a1+a2+…+ak < a1.a2.….ak and a1+a2+…+ak > a1.a2.….ak respectively.

Any typical element in $P(X1)\times P(X2)\times\ldots\times P(Xn)$ is of the form $(A1,A2,\ldots,An)$ where $Ai\subseteq Xi$ for $i\in\{1,2,\ldots,n\}$. Suppose $(A1,A2,\ldots,An)$ and $(B1,B2,\ldots,Bn)$ are any two members in $P(X1)\times P(X2)\times\ldots\times P(Xn)$. Throughout this chapter we use the following notations and terminologies.

$(X1, X2,\ldots,Xn)$ is an n-ary absolute set and $(\varnothing, \varnothing, \varnothing,\ldots,\varnothing)$ is an n-ary null set or void set or empty set in $P(X1)\times P(X2)\times\ldots\times P(Xn)$. $(A1,A2,\ldots,An)\subseteq(B1, B2,\ldots,Bn)$ if $Ai\subseteq Bi$ for every $i\in\{1,2,\ldots,n\}$ and $(A1,A2,\ldots,An) \neq (B1, B2,\ldots,Bn)$ if $Ai\neq Bi$ for some $i\in\{1,2,\ldots,n\}$. Equivalently $(A1,A2,\ldots,An)=(B1, B2,\ldots,Bn)$ if $Ai=Bi$ for every $i\in\{1,2,\ldots,n\}$. If $Ai\neq Bi$ for each $i\in\{1,2,\ldots,n\}$ then we say $(A1,A2,\ldots,An)$ is absolutely not equal to $(B1, B2,\ldots,Bn)$ which is denoted as $(A1,A2,\ldots,An)$ a$\neq$ $(B1, B2,\ldots,Bn)$. Let $xi\in Xi$ and $Ai\subseteq Xi$ for every $i\in\{1,2,\ldots,n\}$. Then $(x1, x2,\ldots,xn)\in(A1, A2,\ldots, An)$ if $xi\in Ai$ for every $i\in\{1,2,\ldots,n\}$.

2.7. Definition-

Let Xi be an infinite set for every $i\in\{1,2,\ldots,n\}$.

$(A1, A2, \ldots, An)$ is finite if Ai is finite for every $i\in\{1,2,\ldots,n\}$ and is infinite if Ai is infinite for some $i\in\{1,2, ,\ldots,n\}$.

2.8. Definition-

Let Xi be an uncountable set for every $i\in\{1, 2,\ldots,n\}$,

$(A1, A2,\ldots,An)$ is countable if Ai is countable for every $i\in\{1,2,\ldots,n\}$ and is uncountable if Ai is uncountable for some $i\in\{1,2,\ldots,n\}$.

2.9. Proposition-

$(x1, x2,\ldots,xn)\in(A1, A2,\ldots,An)$ iff $(x1, x2,\ldots,xn) \in A1\times A2\times\ldots\times An$.

The notions of n-ary union, n-ary intersection, n-ary complement and n-ary difference of n-ary sets are defined component wise. Two n-ary sets are said to be n-ary disjoint if the sets in the corresponding positions are disjoint and $(A1, A2, \ldots, An)$ is a somewhat empty n-ary set if $Ai \neq\varnothing$ for at least one $i\in\{1,2,\ldots,n\}$ and $Aj =\varnothing$ for at least one $j\in\{1,2,\ldots,n\}$.

Let S denote the collection of all somewhat empty n-ary sets in $P(X1)\times P(X2)\times\ldots\times P(Xn)$. Let N $P(X1)\times P(X2)\times\ldots\times P(Xn) \setminus S$, the collection of all n-ary sets other than somewhat empty n-ary sets. The next proposition shows that N can be considered as a proper subset of $P(X1\times X2\times\ldots\times Xn)$.

2.10. Proposition-

Let φ: N $\rightarrow P(X1\times X2\times\ldots\times Xn)$ be defined by $\varphi(A1,A2,\ldots,An) =A1\times A2\times\ldots\times An$ for each element $(A1,A2,\ldots,An)$ in $P(X1)\times P(X2)\times\ldots\times P(Xn)$. Then φ is injective but not surjective.

Proof: Suppose $(A1, A2,\ldots,An)$ and $(B1, B2,\ldots,Bn)$ are any two distinct members of M. Then $Ai\neq Bi$ for some $i\in\{1,2,\ldots,n\}$ that implies $A1\times A2\times\ldots\times An \neq B1\times B2\times\ldots\times Bn$. Therefore

$\varphi(A1, A2,\ldots,An) \neq \varphi(B1, B2,\ldots,Bn)$ that implies φ is injective. Further φ is not surjective as shown below.

$X1=\{a, b, c\}$ and $X2=\{1, 2\}$, $A1\subseteq X1$ and $A2\subseteq X2$. Let $S=\{(a, 1), (b, 2)\}\subseteq X1\times X2$. It can be seen that there is no $(A1, A2)\in P(X1\times X2)$ such that $A1\times A2=S$.

2.11. Remark-

The function φ, defined above is not injective if we replace M by $P(X1)\times P(X2)\times\ldots\times P(Xn)$.

Let $f:Y\rightarrow X1\times X2\times\ldots\times Xn$ be a single valued function. Then it induces a function

f-1: $P(X1)\times P(X2)\times\ldots\times P(Xn)\rightarrow P(Y)$ that is an n-ary set to set valued function defined by $f\text{-}1((A1, A2,\ldots,An))=\{y:f(y)\in(A1, A2,\ldots,An)\}=\{y: pi(f(y))\in Ai$ for each $i\in\{1,2,\ldots,n\}\}$ where each pi is a projection of $X1\times X2\times\ldots\times Xn$ onto Xi .

2.12. Proposition-

Let $f:Y\rightarrow X1\times X2\times\ldots\times Xn$ be a single valued function. Then

$f\text{-}1((A1, A2,\ldots,An))= f\text{-}1(A1\times A2\times\ldots\times An)$ for every $(A1, A2,\ldots,An) \in M$.

Proof: $f\text{-}1((A1, A2,\ldots, An)) =\{y: f(y)\in(A1, A2,\ldots,An)\}$

$=\{y: pi(f(y))\in Ai$ for each $i\in\{1,2,3,\ldots,n\} \} = \{y: f(y)\in A1\times A2\times\ldots\times An\}$

$= f^{-1}(A_1 \times A_2 \times \ldots \times A_n)$.

2.13. Proposition-

Let $f: Y \to X_1 \times X_2 \times \ldots \times X_n$ be a single valued function. Then f^{-1} preserves n-ary union and n-ary intersection.

2.14. Proposition-

$P(X_1) \times P(X_2) \times \ldots \times P(X_n)$ is a complete distributive lattice under n-ary set inclusion relation.

III. N-ARY TOPOLOGY-

Let $X_1, X_2, X_3, \ldots, X_n$ be the non empty sets. Let $T \subseteq P(X_1) \times P(X_2) \times \ldots \times P(X_n)$.

3.1. Definition-

T is an n-ary topology on $(X_1, X_2, X_3, \ldots, X_n)$ if the following axioms are satisfied.

(i). $(\varnothing, \varnothing, \varnothing, \ldots, \varnothing) \in T$

(ii). $(X_1, X_2, X_3, \ldots, X_n) \in T$

(iii) If $(A_1, A_2, \ldots, A_n), (B_1, B_2, \ldots, B_n) \in T$ then

$(A_1, A_2, \ldots, A_n) \cap (B_1, B_2, \ldots, B_n) \in T$

(iv) If $(A_{1\alpha}, A_{2\alpha}, \ldots, A_{n\alpha}) \in T$ for each $\alpha \in \Omega$ then $\bigcup_{\alpha \in \Omega} (A_{1\alpha}, A_{2\alpha}, \ldots, A_{n\alpha}) \in T$.

If T is an n-ary topology then the paire (X, T) is called an n-ary topological space. The elements $(x_1, x_2, \ldots, x_n) \in X_1 \times X_2 \times \ldots \times X_n$ are called the n-ary points of (X, T) and the members $(A_1, A_2, \ldots, A_n)$ of $P(X_1) \times P(X_2) \times \ldots \times P(X_n)$. are called the n-ary sets of (X, T). The members of T are called the n-ary open sets in (X, T). It is noteworthy to see that product topology on $X_1 \times X_2 \times \ldots \times X_n$ and n-ary topology on $(X_1, X_2, X_3, \ldots, X_n)$ are independent concepts as any open set in product topology is a subset of $X_1 \times X_2 \times X_3 \times \ldots \times X_n$ and an open set in an n-ary topology is a member of $P(X_1) \times P(X_2) \times \ldots \times P(X_n)$.

3.2. Examples-

(i). $I = \{ (\varnothing, \varnothing, \varnothing, \ldots, \varnothing), (X_1, X_2, \ldots, X_n) \}$ is an n-ary topology, called indiscrete n-ary topology .

(ii). $D = P(X_1) \times P(X_2) \times \ldots \times P(X_n)$ is an n-ary topology, called discrete n-ary topology .

(iii). $\{(\varnothing, \varnothing, \varnothing, \ldots, \varnothing)\} \cup \{ A: a=(a_1, a_2, a_3, \ldots, a_n) \in (A_1, A_2, A_3, \ldots, A_n) \}$ is an n-ary topology, called n-ary point inclusion n-ary topology.

(iv). $\{(\varnothing, \varnothing, \varnothing, \ldots, \varnothing)\} \cup \{A : B \subseteq A \}$ is an n-ary topology, called n-ary set inclusion n-ary topology.

(v). $\{ (\varnothing, \varnothing, \varnothing, \ldots, \varnothing), (A_1, A_2, A_3, \ldots, A_n), (X_1 \backslash A_1, X_2 \backslash A_2, \ldots, X_n \backslash A_n), X \}$ is an n-ary topology .

(vi). If $B \subseteq A$ then $\{(\varnothing, \varnothing, \varnothing, \ldots, \varnothing), B, A, (X_1, X_2, X_3, \ldots, X_n) \}$ is an n-ary topology.

(vii). If $\{(\varnothing, \varnothing, \varnothing, \ldots, \varnothing), A, (X_1, X_2, X_3, \ldots, X_n) \}$ is an n-ary topology.

(viii). $Diag(X) = \{\{ (A_1, A_1, \ldots, A_1) : A_1 \subseteq X_1 \}$ is an n-ary topology .

(ix). $F = \{(\varnothing, \varnothing, \varnothing, \ldots, \varnothing)\} \cup \{ A : (X_1 \backslash A_1, X_2 \backslash A_2, \ldots, X_n \backslash A_n)$, is finite $\}$ is an n-ary topology called co-finite n-ary topology.

(x). $C = \{(\varnothing, \varnothing, \varnothing, \ldots, \varnothing)\} \cup \{ A: (X_1 \backslash A_1, X_2 \backslash A_2, \ldots, X_n \backslash A_n),,$ is countable $\}$ is an n-ary topology called co-countable n-ary topology.

(xi). $\{X\} \cup \{ A : (a_1, a_2, a_3, \ldots, a_n) \notin A\}$ is an n-ary topology, called n-ary point exclusion n-ary topology.

(xii). Let R= the set of all real numbers and R^n = the Cartesian product of n-copies of R. Let $a=(a_1, a_2, \ldots, a_n) \in (R, R, \ldots, R)$. Then for each $r=(r_1, r_2, \ldots, r_n) >0$. Let $S(a, r)=(S(a_i, r_i),, \ldots, S(a_n, r_n))$ where $S(a_i, r_i)=\{x_i: |x_i-a_i|<r_i\}$ for $i \in \{1,2,3,..,n\}$. Let E = the set of all n-ary sets $(A_1, A_2, \ldots, A_n) \in P(R) \times P(R) \times \ldots \times P(R)$ such that for every $(a_1, a_2, \ldots, a_n) \in (A_1, A_2, \ldots, A_n)$ there exists $(r_1, r_2, \ldots, r_n) >0$ such that

$S(a_i, r_i) \subseteq A_i$ for $i \in \{1,2,3,..,n\}$. Then E is an n-ary topology, called n-ary Euclidean topology over R.

The next proposition is easy to establish.

3.3. Proposition-

Let $f: Y \to X_1 \times X_2 \times \ldots \times X_n$ be a single valued function. Let T be an n-ary topology on X. Then $f^{-1}(T) = \{f^{-1}((A_1, A_2, A_3, \ldots, A_n)): (A_1, A_2, A_3, \ldots, A_n) \in T \}$ is a topology on Y.

3.4. Proposition-

Let $f_i:Y\rightarrow X_i$ be a single valued function for every $i\in\{1,2,3,\ldots,n\}$. Let $f = (f_1, f_2,\ldots,f_n):Y\rightarrow X_1\times X_2\times\ldots\times X_n$ be defined by $f(y) = (f_1(y), f_2(y),\ldots,f_n(y))$ for every $y\in Y$ where $f_i(y) = p_i(f(y))$ for every $y\in Y$. Let τ be a topology on Y. Then if f is a bijection then $\{f(A): A\in\tau\}$ is an n-ary topology on $(X_1, X_2,\ldots,X_n)$.

Proof: Let A be a subset of Y and $y\in A$ with $f(y)\in f(A)$.Then
$f(y) = (f_1(y), f_2(y),\ldots,f_n(y))\in(f_1(A), f_2(A),\ldots,f_n(A))$. Conversely let
$(x_1, x_2,\ldots,x_n)\in(f_1(A), f_2(A),\ldots,f_n(A))$ that implies $x_i=f_i(y)=p_i(f(y))$ for some $y\in A$, $i\in\{1,2,3,\ldots,n\}$. That is $(x_1, x_2,\ldots,x_n)\in f(A)$. Therefore $f(A)=(f_1(A), f_2(A),\ldots,f_n(A))\in P(X_1)\times P(X_2)\times\ldots\times P(X_n)$. Since f is a bijection, each f_i is also a bijection. Therefore it is easy to verify that $\{f(A):A\in\tau\}$ is an n-ary topology on $(X_1, X_2,\ldots,X_n)$.

3.5. Proposition-

Let p be a permutation of $(1,2,,\ldots,n)$ defined by $p(i)=p_i$, $i\in\{1,2,3,\ldots,n\}$. Let T be an n-ary topology on $(X_1, X_2,\ldots,X_n)$. $p(T) =\{(A_{p(1)}, A_{p(2)},\ldots,A_{p(n)}): (A_1, A_2,\ldots,A_2)\in T\}$ is an n-ary topology on $(X_{p(1)}, X_{p(2)},\ldots,X_{p(n)})$.

Proof: Follows from the fact that $(A_{p(1)}, A_{p(2)},\ldots,A_{p(n)})\in P(X_{p(1)})\times P(X_{p(2)})\times\ldots\times P(X_{p(n)})$ iff $(A_1, A_2,\ldots,A_2)\in P(X_1)\times P(X_2)\times\ldots\times P(X_n)$.

The next two propositions can be proved easily.

3.6. Proposition-

Let T be an n-ary topology on $(X_1, X_2,\ldots,X_n)$. $T_1 =\{A_1: (A_1, A_2,\ldots,A_2)\in T\}$ and T_2, T_3, …, T_n can be similarly defined. Then T_1, T_2 ,, …, T_n are topologies on $X_1, X_2, \ldots,X_n$ respectively.

3.7. Proposition-

Let $\tau_1, \tau_2, \ldots,\tau_n$ be the topologies on $X_1, X_2, \ldots,X_n$ respectively. Let $T = \tau_1\times\tau_2\times\ldots\times\tau_n = \{(A_1, A_2,\ldots,A_2): A_i\in\tau_i\}$. Then T is an n-ary topology on $(X_1, X_2,\ldots,X_n)$. Moreover $T_i = \tau_i$ for every $i\in\{1,2,3,\ldots,n\}$.

IV. CONCLUSION-

The concept of binary topology has been extended to n-ary topology for n>1 sets. The basic properties have been discussed. In particular it is observed that the notions of product topology and n-ary topology are different. More over the connection between the product topology and an n-ary topology is studied.

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