Some Theorems In Anti S-Fuzzy Subfield Of A Field

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Abstract- In this paper, we made an attempt to study the algebraic nature of anti S-fuzzy subfield of a field.

Keywords – S-norm, fuzzy subset, union, anti S-fuzzy subfield.

I. INTRODUCTION

After the introduction of fuzzy sets by L.A.Zadeh [11], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subgroups, anti-fuzzy subgroups, fuzzy fields and fuzzy linear spaces was introduced by Biswas.R [4, 5]. In this paper, we introduce the some theorems in anti S-fuzzy subfield of a field.

II. PRELIMINARIES

2.1 Definition: A S-norm is a binary operation S: $[0, 1]\times[0, 1] \rightarrow [0, 1]$ satisfying the following requirements;

(i) $0 \, S \, x = x$, $1 \, S \, x = 1$ (boundary condition)

(ii) $x \, S \, y = y \, S \, x$ (commutativity)

(iii) $x \, S \, (y \, S \, z) = (x \, S \, y) \, S \, z$ (associativity)

(iv) if $x \leq y$ and $w \leq z$, then $x \, S \, w \leq y \, S \, z$ (monotonicity).

2.2 Definition: Let X be a non-empty set. A fuzzy subset A of X is a function $A : X \rightarrow [0, 1]$.

2.3 Definition: The union of two fuzzy subsets A and B of a set X is defined by $(A\cup B)(x) = \max \{ A(x), B(x) \}$, for all x in X.

2.4 Definition: Let (F, +, ·) be a field. A fuzzy subset A of F is said to be an anti S-fuzzy subfield (anti fuzzy subfield with respect to S-norm) of F if the following conditions are satisfied:

(i) $A(x+y) \leq S(A(x), A(y))$, for all x and y in F,

(ii) $A(- x) \leq A(x)$, for all x in F,

(iii) $A(xy) \leq S(A(x), A(y))$, for all x and y in F,

(iv) $A(x^{-1}) \leq A(x)$, for all x in $F-\{0\}$, where 0 is the additive identity element of F.

III. PROPERTIES OF ANTI S-FUZZY SUBFIELDS

3.1 Theorem: If A is an anti S-fuzzy subfield of a field (F, +, ·), then $A(-x) = A(x)$, for all x in F and $A(x^{-1}) = A(x)$, for all x in $F-\{0\}$ and $A(x) \geq A(0)$, for all x in F and $A(x) \geq A(1)$, for all x in F, where 0 and 1 are identity elements in F.

Proof: For x in F and 0, 1 are identity elements in F. Now, $A(x) = A(-(-x)) \leq A(-x) \leq A(x)$. Therefore, $A(-x) = A(x)$, for all x in F. And, $A(x) = A((x^{-1})^{-1}) \leq A(x^{-1}) \leq A(x)$. Therefore, $A(x^{-1}) = A(x)$, for all x in $F-\{0\}$. And, $A(0) = A(x-x) \leq S(A(x), A(-x)) = A(x)$. Therefore, $A(0) \leq A(x)$, for all x in F. And, $A(1) = A(xx^{-1}) \leq S(A(x), A(x^{-1})) = A(x)$. Therefore, $A(1) \leq A(x)$, for all x in F.

3.2 Theorem: If A is an anti S-fuzzy subfield of a field (F, +, ·), then (i) $A(x-y) = A(0)$ gives $A(x) = A(y)$, for all x and y in F,

(ii) $A(xy^{-1}) = A(1)$ gives $A(x) = A(y)$, for all x and $y \neq 0$ in F, where 0 and 1 are identity elements in F.

Proof: Let x and y in F and 0, 1 are identity elements in F. (i) Now, $A(x) = A(x-y+y) \leq S(A(x-y), A(y)) = S(A(0), A(y)) = A(y) = A(x-(x-y)) \leq S(A(x-y), A(x)) = S(A(0), A(x)) = A(x)$. Therefore, $A(x) = A(y)$, for all x and y in F. (ii) Now, $A(x) = A(xy^{-1}y) \leq S(A(xy^{-1}), A(y)) = S(A(1), A(y)) = A(y) = A((xy^{-1})^{-1}x) \leq S(A(xy^{-1}), A(x)) = S(A(1), A(x)) = A(x)$. Therefore, $A(x) = A(y)$, for all x and $y\neq 0$ in F.

3.3 Result: A is an anti S-fuzzy subfield of a field (F, +, ·) if and only if $A(x-y) \leq S(A(x), A(y))$, for all x and y in F and $A(xy^{-1}) \leq S(A(x), A(y))$, for all x and $y \neq 0$ in F.

3.4 Theorem: Let A be a fuzzy subset of a field (F, +, ·). If $A(e) = A(e^1) = 0$, $A(x−y) \le S(A(x), A(y))$, for all x and y in F and $A(xy^{-1}) \le S(A(x), A(y))$, for all x and y ≠ e in F, then A is an anti S-fuzzy subfield of F, where e and e^1 are identity elements of F.
Proof: It is trivial.
3.5 Theorem: If A is an anti S-fuzzy subfield of a field (F, +, ·), then H = { x / x∈F: A(x) = 0 } is either empty or is a subfield of F.
Proof: If no element satisfies this condition, then H is empty. If x and y in H, then $A(x−y) \le S(A(x), A(−y)) = S(A(x), A(y)) = S(0, 0) = 0$. Therefore, $A(x−y) = 0$, for all x and y in F. We get x−y in H. And, $A(xy^{-1}) \le S(A(x), A(y^{-1})) = S(A(x), A(y)) = S(0, 0) = 0$. Therefore, $A(xy^{-1}) = 0$, for all x and y≠ e in F. We get xy^{-1} in H. Therefore, H is a subfield of F. Hence H is either empty or is a subfield of F.
3.6 Theorem: If A is an anti S-fuzzy subfield of a field (F, +, ·), then H ={ x∈F: $A(x) = A(e) = A(e^1)$ } is either empty or is a subfield of F, where e and e^1 are identity elements of F.
Proof: If no element satisfies this condition, then H is empty. If x and y satisfies this condition, then $A(−x) = A(x) = A(e)$, for all x in F and $A(x^{-1}) = A(x) = A(e^1)$, for all x in F−{e}. Therefore, $A(−x) = A(e)$, for all x in F and $A(x^{-1}) = A(e^1)$, for all x in F−{e}. Hence −x , x^{-1} in H. Now, $A(x−y) \le S(A(x), A(−y)) \le S(A(x), A(y)) = S(A(e), A(e)) = A(e)$. Therefore, $A(x−y) \le A(e)$ ------- (1). And, $A(e) = A((x−y)−(x−y)) \le S(A(x−y), A(−(x−y))) \le S(A(x−y), A(x−y)) = A(x−y)$. Therefore, $A(e) \le A(x−y)$ --------(2). From (1) and (2), we get $A(e) = A(x−y)$, for all x and y in F. Now, $A(xy^{-1}) \le S(A(x), A(y^{-1})) \le S(A(x), A(y)) = S(A(e^1), A(e^1)) = A(e^1)$. Therefore, $A(xy^{-1}) \le A(e^1)$ ------------- (3). And, $A(e^1) = A((xy^{-1})(xy^{-1})^{-1}) \le S(A(xy^{-1}), A((xy^{-1})^{-1})) \le S(A(xy^{-1}), A(xy^{-1})) = A(xy^{-1})$. Therefore, $A(e^1) \le A(xy^{-1})$ ------------ (4). From (3) and (4), we get $A(e^1) = A(xy^{-1})$, for all x and y ≠ e in F. Hence $A(e) = A(x−y)$, $A(e^1) = A(xy^{-1})$. We get x−y, xy^{-1} in H. Hence H is either empty or is a subfield of F.
3.7 Theorem: Let A be an anti S-fuzzy subfield of a field (F, +, ·). Then (i) if $A(x−y) = 0$, then $A(x) = A(y)$, for x and y in F (ii) if $A(xy^{-1}) = 0$, then $A(x) = A(y)$, for all x and y≠ e in F, where e and e^1 are identity elements of F.
Proof: Let x and y in F. Now, $A(x) = A(x−y+y) \le S(A(x−y), A(y)) = S(0, A(y)) = A(y) = A(−y) = A(−x+x−y) \le S(A(−x), A(x−y)) = S(A(−x), 0) = A(−x) = A(x)$. Therefore, $A(x) = A(y)$, for all x, y in F. And, $A(x) = A(xy^{-1}y) \le S(A(xy^{-1}), A(y)) = S(0, A(y)) = A(y) = A(y^{-1}) = A (x^{-1}xy^{-1}) \le S(A(x^{-1}), A(xy^{-1})) = S(A(x^{-1}), 0) = A(x^{-1}) = A(x)$. Therefore, $A(x) = A(y)$, for all x and y≠ e in F.
3.8 Theorem: If A is an anti S-fuzzy subfield of a field (F, +, ·) , then (i) if $A(x−y) =1$, then either $A(x) = 1$or $A(y) = 1$, for x and y in F, (ii) if $A(xy^{-1}) = 1$, then either $A(x) = 1$or $A(y) = 1$, for all x and y≠ e in F.
Proof: Let x and y in F. By the definition $A(x−y) \le S(A(x), A(y))$, which implies that $1\le S(A(x), A(y))$. Therefore, either $A(x) = 1$or $A(y) = 1$, for all x and y in F. And by the definition $A(xy^{-1}) \le S(A(x), A(y))$, which implies that $1 \le S(A(x), A(y))$. Therefore, either $A(x) = 1$or $A(y) = 1$, for all x and y≠ e in F.
3.9 Theorem: Let (F, +, ·) be a field. If A is an anti S-fuzzy subfield of F, then $A(x+y) = S(A(x), A(y))$, for all x and y in F and $A(xy) = S(A(x), A(y))$, for all x and y in F with $A(x) \ne A(y)$.
Proof: Let x and y belongs to F. Assume that $A(x) < A(y)$. Now, $A(y) = A(−x+x+y) \le S(A(−x), A(x+y)) \le S(A(x), A(x+y)) = A(x+y) \le S(A(x), A(y)) = A(y)$. Therefore, $A(x+y) = A(y) = S(A(x), A(y))$, for all x and y in F. And, $A(y) = A(x^{-1}xy) \le S(A(x^{-1}), A(xy)) \le S(A(x), A(xy)) = A(xy) \le S(A(x), A(y)) = A(y)$. Therefore, $A(xy) = A(y) = S(A(x), A(y))$, for all x and y in F.
3.10 Theorem: If A and B are any two anti S-fuzzy subfields of a field (F, +, ·), then their union A∪B is an anti S-fuzzy subfield of F.
Proof: Let x and y belongs to F and A = { ⟨ x, A(x) ⟩ / x∈F } and B = { ⟨ x, B(x) ⟩ / x∈F }. Let C = A∪B and C = { ⟨ x, C(x) ⟩ / x∈F }, C(x) = max { A(x), B(x) }. (i) C(x−y) = max { A(x−y), B(x−y) } ≤ max { S(A(x), A(y)), S(B(x), B(y)) } = S(max { A(x), B(x) }, max{A(y), B(y)}) = S(C(x), C(y)). Therefore, C(x−y) ≤ S(C(x), C(y)), for all x and y in F. (ii) $C(xy^{-1})$ = max { $A(xy^{-1})$, $B(xy^{-1})$ } ≤ max { S(A(x), A(y)), S(B(x), B(y)) } = S (max { A(x), B(x) }, max { A(y), B(y) }) = S(C(x), C(y)). Therefore, $C(xy^{-1}) \le S(C(x), C(y))$, for all x and y ≠ 0 in F. Hence A∪B is an anti S-fuzzy subfield of a field F.
3.11 Theorem: The union of a family of anti S-fuzzy subfields of a field (F, +, ·) is an anti S-fuzzy subfield of F.
Proof: It is trivial.

IV. REFERENCES

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