A method to Graphically depict the Standard Deviations of Astigmatism Vectors on Single Angle Polar Plot

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Abstract- This paper discusses mathematics of single and double angle plots used to study astigmatism data. Literature reported advantages and disadvantages of the types of plots are summarized. A method to graphically show standard deviation hemi-ellipse of astigmatism data on a single angle polar plot is proposed and its advantages and disadvantages are discussed.

I. INTRODUCTION

Astigmatism is a common refractive error of the eye that most commonly occurs due to ellipsoid shape of cornea, that is unequal curvature of cornea along the two orthogonal axes. Astigmatism is a directional measure with magnitude and axis.[1-4] Studying astigmatism in a group of patients or in a population needs use of statistical methods. As such, astigmatism must be transformed to rectangular vector from the conventional scalar.

II. DOUBLE-ANGLE VECTOR PLOT

Double angle vector plots are commonly used to represent success of astigmatism vector after surgical management of astigmatism. Mathematically, direction of vectors rotates 360º before returning to its origin. In eye refraction, astigmatism vector returns to its initial value after 180º. To mathematically study the astigmatism, double angle vector plots convert the astigmatism data to cartesian coordinates with double the astigmatism axis. This converts the 180º astigmatism data to mathematical 360º vector plots 'double-angle vector plots'. To keep the orientation of 180º for eyecare professionals, a geometric angle of 90º is labeled as 45º, 180º is labeled as 90º, 270º is labeled as 135º and 0º/360º is labeled as 0º/180º.

Double angle vector plot displays not only the centroid but also the confidence ellipse (Figure 1). Centroid represents the average of x coordinates and y coordinates, thus accounting for both the magnitude and direction when analyzing astigmatism vectors for a sample/population. Confidence ellipse displays the standard deviations (SDs) of the x and y coordinates and is displayed as an ellipse surrounding the centroid. The orientation of the confidence ellipse also provides some insights. When most of the data in a population are oriented with-the-rule (WTR, ophthalmologically 60º-120º) or against-the-rule (ATR) (0º-30º, 150º-180º), the major axis of the ellipse is oriented horizontally. In contrast, when majority of data have oblique astigmatism (30º-60º, 120º-150º), major axis of the ellipse would be oriented vertically. In a population with equal distribution of oblique and non-oblique (WTR/ATR) astigmatism, the standard deviation ellipse would be circular.

Figure 1 represents hypothetical data of patients that have undergone an astigmatism correcting procedure. Preoperative and postoperative individual data points are represented in orange and green circles, respectively. Preoperative centroid (red diamond) and horizontally oriented, large ellipse (red) shows that most of the astigmatism data are either WTR or ATR. Postoperative centroid (blue diamond) and horizontally oriented, small ellipse (blue) show that most of the postoperative astigmatism data were still either WTR or ATR, but smaller in magnitude. If there was a change in WTR/ATR astigmatism to oblique, orientation of the post-operative ellipse would have been vertical. Since the ellipses of both WTR and ATR astigmatism would show up horizontally on double angle vector plot, it is not possible to assess from the ellipses if treatment changed from WTR preoperative to ATR postoperatively or vice versa.

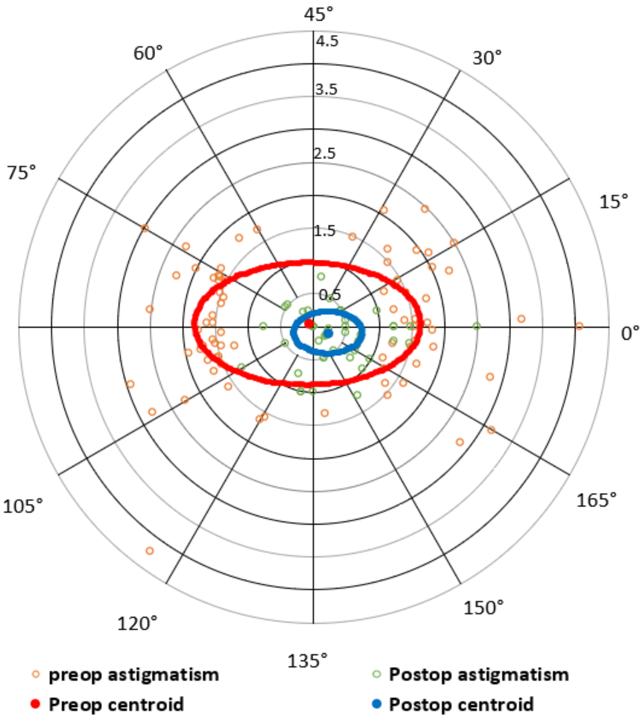

Figure 1: Double angle vector plot of hypothetical data of patients that have undergone an astigmatism correcting procedure showing the SDs of x and y coordinates as an ellipse.

III. SINGLE-ANGLE POLAR PLOT

Single angle polar plots also help study astigmatism change in a group of patients/population (Figure 2). In contrast to the doubling of angle with double angle plots, single angle polar plots use single angle (ophthalmologically, 0º-180º) to calculate x and y coordinates and display in a 0º-180º hemicircle. Vectoral mean of single angle polar plots show the summated vector mean of a bunch of astigmatism data (calculated using single angle 0º-180º). Eye care professionals may relate to single-angle polar plots much more easily as it displays astigmatism information in a manner that is quite similar to what they routinely practice in their clinics. As such, they can directly use the data presented on a single angle polar plot to the clinical situation of astigmatism diagnostics and therapeutics.

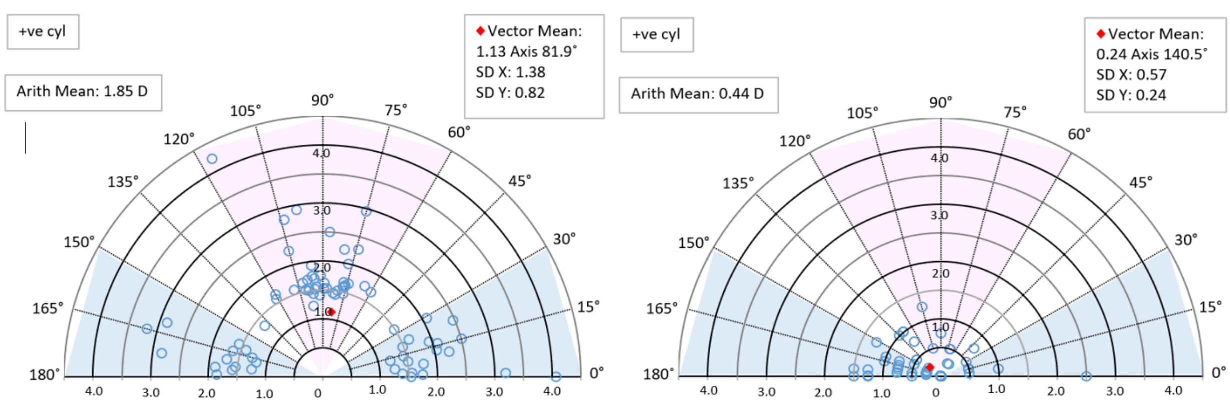

Figure 2: Single angle polar plots (a) preoperative astigmatism and (b) postoperative astigmatism.

It is recommended that the single angle polar plots be used only for graphical display purposes. These do not amount to vector calculations in a 0° to 180° space. When needed, vector calculations must be performed after doubling the angle to convert the astigmatism data into 360° Cartesian coordinates vectors so that standard vector mathematics can apply. Single-angle polar plots do not show the standard deviation confidence ellipse.6 Instead the values of X and Y standard deviations are presented numerically in a box. As discussed above, the double-angle plots are advantageous in a manner that it visually groups data points at 0° and 180° together. Single angle polar plots, in contrast, plot them on the opposite sides. While this may be mathematically confusing, eye care professionals relate with single-angle polar plots very well and easily recognize that points near 0° are vectorially similar to the points near 180° (ATR). To allow better orientation of ATR axes. regions corresponding to the ATR axes are shaded in same color to indicate areas of common orientation.

IV. A METHOD TO GRAPHICALLY DEPICT THE STANDARD DEVIATIONS ON A SINGLE ANGLE POLAR PLOT

While the single and double angle vector plots have their respective advantages and disadvantages, adding visual depiction of the standard deviation of x and y coordinates, something similar to the confidence ellipse of double angle vector plot, may allow better visual comparison between two groups. Since single polar plot show astigmatic data in 0°-180° space, the author proposes that to demonstrate SD of x & y coordinates as a hemi-ellipse. When majority of astigmatism data in a sample are oriented ATR (0°-45° 135°-180°), semi-major axis of such a hemi-ellipse will be oriented horizontally. In contrast, when high proportion of the astigmatism data in a sample are oriented WTR (45°-135°), semi-major axis of such a hemi-ellipse will be oriented vertically.

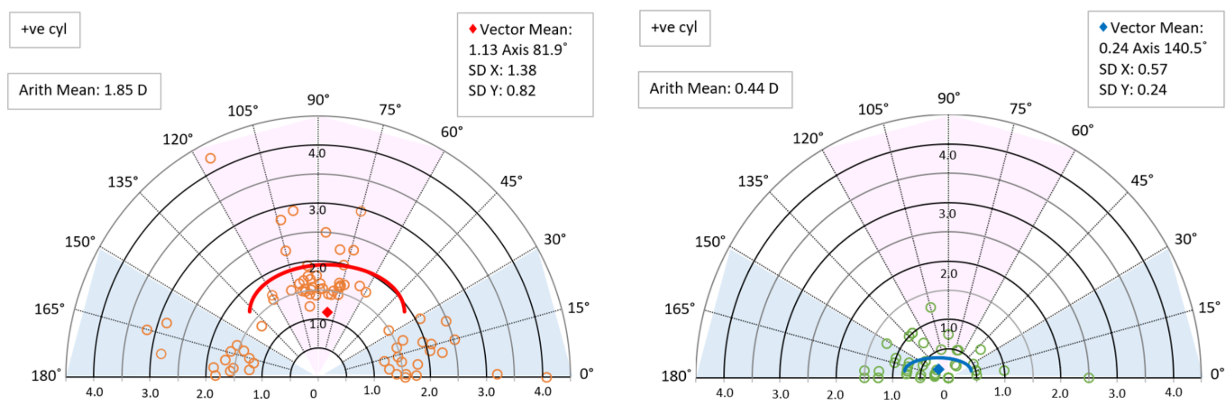

Figure 3: Single angle polar plots (a) preoperative astigmatism and (b) postoperative astigmatism displaying the SDs of x and y coordinates as a hemi-ellipse. Color scheme chosen in these single angle polar plots is similar to Figure 1 displaying double angle vector plot.

Unlike the ellipse of a double angle vector plot, overall oblique orientation (30º-60º, 120º-150º) of a group of data cannot be represented on such a hemi-ellipse. This hemi-ellipse, however, will be able to tell if there was flipping of preoperative astigmatism axis in these group of patients (representing both the magnitude and direction of astigmatism). From the comparison of orientation of double angle vector ellipse of preoperative and post-operative data (Figure 1), it was not possible to comment if there was flipping of axis (ATR preoperatively - WTR postoperatively or vice versa) in these group of eyes as a whole (magnitude as well as direction of astigmatism). Comparison of the hemi-ellipses of preoperative and postoperative single angle polar plots (Figure 3) however can tell that majority of the astigmatism data (magnitude and direction) were aligned ATR pre-operatively and remained so post-operatively, as semi-major axis of both the pre and postoperative data are aligned horizontally; as such there was no flipping of axis to WTR (45°-135°) in this group of eyes. Inclusion of the hemi-ellipse to display the SDs of the x and y coordinates around the vectoral mean may provide single angle polar plots with a visual depiction of the spread of the data when comparing preoperative and postoperative astigmatism data.

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